

LOGIC

PART FIRST.

DEDUCTION

BY

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P R E F A C E.

THE present work aims at embracing a full course of Logic, both Formal and Inductive.

In an introductory chapter, are set forth such doctrines of psychology as have a bearing on Logic, the nature of knowledge in general, and the classification of the sciences; the intention being to avoid doctrinal digressions in the course of the work. Although preparatory to the understanding of what follows, this chapter may be passed over lightly on a first perusal of the work.

The part on Deduction contains the usual doctrines of the Syllogism, with the additions of Hamilton, and a full abstract of the novel and elaborate schemes of De Morgan and Boole.

The Inductive portion comprises the methods of inductive research, and all those collateral topics brought forward by Mr. Mill, as part of the problem of Induction; various modifications being made in the manner of statement, the order of topics, and the proportion of the handling. The greatest innovation is the rendering of Cause by the new doctrine called the Conservation, Persistence, or Correlation of Force.

Mr. Mill's view of the relation of Deduction and Induction is fully adopted, as being the solution of the otherwise inextricable puzzle of the syllogism, and the means of giving unity and comprehensiveness to Logic.

A separate division is appropriated to the Logic of the Sciences, with the view of still further exemplifying the logical methods, and of throwing light upon various points in the sciences themselves. The review comprises all the theoretical or fundamental sciences—Mathematics, Physics, Chemistry, Biology, and Psychology; the sciences of Classification, or Natural History; and two leading Practical sciences—Politics and Medicine.

The department of Definition is, for the first time, brought under a methodical scheme, and rendered of co-ordinate value with Deduction and Induction, as a branch of logical method. The modes of defining, as a generalizing process, are given under two canons, a positive and a negative; and attention is called to the chief obstacles—uncertainty in the denotation of words, and the gradual transition of qualities into their opposites.

In discussing Fallacies, I have canvassed the grounds for the usual practice of detaching the violations of logical rules from the exposition of the rules themselves; and have endeavoured to show that the only portions of the subject proper to reserve for separate handling, are the Fallacious tendencies of the Mind, and Fallacies of Confusion. As these are subjects of great moment, and admit of wide illustration, both are considered with some minuteness.

None of the controversies in the subject are overlooked; but it has been deemed advisable to separate them from the main body of the work. In an Appendix, are embraced the various Classifications of the Sciences, the Province of Logic, the Classification of Nameable Things, the Universal Postulate, the meanings of Analysis and Synthesis, the Theories of Induction, the Art of Discovery, and the maxims of Historical Evidence.

To adapt the work to an elementary course of Logic,

the parts to be omitted are the Additions to the Syllogism; the Logic of the Sciences, and the chapters in the Appendix. * The junior student, or the candidate for a pass examination, without attempting to master or commit these reserved portions, might yet find their perusal of service in understanding the rest.

There is a general conviction that the utility of the purely Formal Logic is but small; and that the rules of Induction should be exemplified even in the most limited course of logical discipline. I would suggest that an increased attention should be bestowed on Definition and Classification, with reference both to scientific study and to matters not ordinarily called scientific.

As I may be open to the charge of presumption in appearing as a rival to Mr. Mill, I will venture the remark, that an attempt to carry out still more thoroughly the enlarged scheme of logical method, seems the one thing hitherto wanting to the success of his great work.

ABERDEEN, *March, 1870.*

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INTRODUCTION.

1. **LOGIC** may be briefly described as a body of doctrines and rules having reference to Truth.

The functions of **Logic** will be afterwards given with particularity and precision. For the present we remark that it concerns the Truth of things, no matter what the subject be. While in one aspect it is theoretical, in the prevailing aim it is practical.

In this introductory chapter we are to consider the following topics.

- (1) The Psychological data or groundwork of **Logic**.
- (2) The First Principles of **Logic**.
- (3) The Classification of the Sciences.
- (4) The different views of the Province of **Logic**.
- (5) The Divisions of **Logic**.

PSYCHOLOGICAL DATA OF LOGIC.

2. **Logic**, under every view, involves frequent references to the laws and workings of the mind; and the more so the more we extend its province.

In the common **Logic** of the Schools, the **Syllogistic** or **Deductive Logic**, explanations are usually given of the intellectual processes named **Perception** or **Simple Apprehension**, **Abstraction** or the formation of concepts or notions, **Judgment** or the laying down of propositions, and **Reasoning** or the drawing of inferences or conclusions from premises. •

In the **Inductive Logic**, an enquiry is instituted into our

idea of Cause; in connection with which, notice is taken of the controversy respecting the Origin of our Knowledge in the Mind, namely, as to whether it be wholly derived from experience, or whether any portion of it (as Cause, the Axioms of Mathematics, &c.) be intuitive, instinctive, or innate.

It is considered a part of Logic to set forth the theory and the limits of the Explanation of phenomena; for which purpose a reference must be made to the structure of the mental powers. This was the avowed aim of Locke, in his Essay on the Understanding, one of the greatest contributions to the science of mind.

Under such circumstances, the most satisfactory course appears to be to bring forward and expound, once for all, at the commencement, whatever portions of Psychology are in any way implicated with the rules and methods of Logic. But the exposition must necessarily be brief.

Discrimination or Relativity.

3. In order to make us *feel*, there must be a change of impression; whence all feeling is two-sided. This is the law of Discrimination or Relativity.

Observation shows that unbroken continuance of the same impression is attended with unconsciousness; and that the greater the change or transition, the greater the consciousness. An unvarying touch, or a monotonous sound ceases to be felt; in an even temperature, we lose all consciousness of heat or cold. Still more convincing are the instances showing that changes affect us in proportion to their greatness and suddenness. Abrupt transitions are stimulating and exciting; the first exposure to sun-light after being in the dark, the first mouthful of water when we are thirsty, the moment of transition from poverty to wealth—are accompanied with the highest degree of feeling; after which there is a gradual subsidence of the excitement.

Hence the fact of our being under some agency of sense or feeling does not of itself attest our mode of feeling; there must further be given the condition immediately, and for some time previous. That a man is the possessor of a thousand pounds to-day is not a sufficient criterion of his feelings as regards worldly abundance. If a year ago, the same man possessed nothing, he feels in a way totally different from him that has fallen to that amount from a fortune of ten thousand pounds.

4. As regards *Knowledge*, there must likewise be a transition, or change ; and the act of knowing includes always two things.

When we consider our mental states as Knowledge, the same law holds. We know heat by a transition from cold ; light, by passing out of the dark ; up, by contrast to down. There is no such thing as an absolute knowledge of any one property ; we could not know 'motion,' if we were debarred from knowing 'rest.' No one could understand the meaning of a *straight* line, without being shown a line not straight, a bent or crooked line.

We may attend more to one member of the couple than to the other. In this way only can we think of an individual property. We may be thinking more of the heat than of the cold, of the straight than of the crooked ; the one may be the *explicit*, the other the *implicit* subject of our thoughts. As our transitions may be in two directions—from heat to cold, and from cold to heat—we have a difference of feeling in the two cases. We are more conscious of heat, when passing to a higher temperature, and of cold when passing to a lower. The state we have passed to is our *explicit* consciousness, the state we have passed *from* is our *implicit* consciousness.

The principle of Relativity has wide and important bearings in Logic. It will appear in Naming ; in Definition ; in Propositions or Affirmation. It will be appealed to in rectifying a large class of Fallacies—the fallacies of the suppressed relative, or of the Absolute.

Agreement or Similarity.

5. When an impression is repeated, after an interval, we are affected with a new and peculiar consciousness, the shock or consciousness of Agreement in difference.

We see a candle flame ; it is withdrawn ; after a time, it is brought back. We have now, in addition to the luminous effect of the presentation, a shock or feeling of *agreement*, identity, repetition ; a state no less concerned in our intellectual operations than the shock of difference or discrimination. We are constantly experiencing the repetition of former impressions, in circumstances more or less altered, and we are affected with a greater shock according to the greatness of the alteration. The degree or intensity of the consciousness of Agreement may vary through a wide range, from the slight

recognition of a new day to the flash of a great discovery of identification, like Newton's assimilating the fall of a stone to the deflection of the moon towards the earth.

Knowledge as conjoining Difference and Agreement.

6. Our knowledge of a fact is the Discrimination of it from differing facts, and the Agreement or identification of it with agreeing facts.

The only other element in knowledge is the Retentive power of the mind, or memory, which is implied in these two powers.

Our knowledge of heat is (1) a series of shocks of Difference or discrimination between heat and cold, and (2) the Agreements or repetitions of the same shocks under change of circumstances.

Besides the transition heat-cold, which is the primary cognition of heat, we make other transitions into other sensations. We have occasion to pass from a sensation of warmth to a sensation of light, and the difference of the two brings out a new discriminative consciousness, and gives a new meaning to warmth, and also to light; heat is no longer simply the contrast of cold, it is also the contrast of the feeling of luminosity. So, every new sensation that we pass to from heat, with consciousness of difference, gives a new negative meaning to heat; it is not taste, nor smell, nor hardness, nor sound.

Again, our mental impression, knowledge, or idea of a shilling, is the sum of all its differences from the things that we have contrasted it with, and of all its agreements with the things that we have compared it to. We call it round; signifying that it differs from things called square, oblong, oval, &c.; that it *agrees* with other things called round—that we have been frequently struck with the identity of this figure in many different combinations.

So with the weight of the shilling. We know weight by difference, and by agreement; we recognise a shilling as heavier than some things, lighter than others; which is difference; and as identical with a third class, which is agreement.

The knowledge, idea, or recollection of any concrete object, is thus the aggregate of those mental exercises of Discrimination and Agreement, fixed and retained in the mind by the power called retentiveness, or memory; by which power of retention we are able to discriminate and compare

present impressions with past, and to accumulate a vast stock of mental effects or deposits, called ideas, knowledge, thought.

Knowledge is of two kinds, called Object and Subject.

7. The knowledge of a shilling, of a house, of a mountain, of a star, is said to be objective; it relates to the object, or the outer, world. The knowledge of a pleasure or a pain, or of the succession of ideas in the mind, relates to the subject, or the internal, world. We have a great accumulation of both kinds of knowledge; some minds abounding more in one, some more in the other.

Knowledge as (1) Individual and Concrete, or (2) General and Abstract.

8. The knowledge of a table in a room, at a particular time, is in the highest degree individual or concrete. The knowledge relating to any table, at any time, is said to be general and abstract. By the mental power of Agreement or Similarity, we bring to mind different individual tables, attending to their points of community, in spite of many diversities. We affirm properties common to them all. This is the generalising power of the mind. It is one of the most signal functions of our intelligence, and is purely an outgoing of the fundamental power named Agreement, or Similarity.

Dispute as to the Character of General Knowledge, called also Abstract Ideas.

9. In General Knowledge, strictly so called, there is nothing but the fact of agreement among a number of separate particulars; which agreement is signified by the use of a common name.

A general name, as 'circle,' 'round,' 'animal,' 'wise,' is applied to things agreeing in a certain respect, while differing in other respects, to signify their agreement.

It has been supposed that the points of community of agreeing things exist apart from the things. This view is called *Realism*.

It was believed, by a certain school of philosophers, deriving from Plato, that there exists, in the universe of being, a Circle in general, or circular Form without substance, size, or colour; that in like manner, there are archetypal Forms of Man, of

Just, of Good, &c. After a severe controversy, which raged in the scholastic period, this view was abandoned.

Realism is still exemplified, however, in the doctrine of an Independent External World, and also in the doctrine of the separate existence of Mind or Soul. In strictness, the External World is known only as perceived by our senses; Mind is known only as conjoined with body.

Another mode of regarding the fact of community in diversity, is to suppose that the mind can represent to itself in a notion, the points of agreement by themselves; and can leave entirely out of sight the points of difference. This is *Conceptualism*.

Although there is no pure circle in existence, we are supposed able to think of the round figure to the exclusion of the other properties of the individual circles—material, colour, size.

This too is incorrect. It exaggerates the mind's power of giving a preference of attention to some of the attributes of a concrete object, as a wheel, or a shilling. We may think much of the roundness, and little of the size; but we cannot think of the roundness, without thinking of some size or colour.

The usual mode of thinking an abstraction, or of concentrating the mind upon one property, is to think alternately of the different objects possessing the property. We can best think of roundness, by having in view various round things, differing in material, size, colour, &c. The effect of the mind's passing and repassing between the individuals, is that the roundness starts into great prominence, and the other properties fall into the background, without, however, being extinguished. The great fact constantly underlying Abstraction, is the mustering of individuals agreeing in the midst of difference.

We are in the habit of using single individuals to typify a multitude; as in the diagrams of Euclid. We do not, in geometrical reasoning, think of a great number of circular things; we can study the circle upon one figure, provided we take care to *affirm* nothing as to, size, colour, or material, which facts are inseparable even from the barest diagram.

When the logician speaks of a Notion, Concept, or Abstract Idea, he must not be understood as implying anything beyond the agreement of a certain number of things in a given manner.

Our idea of an Individual a conflux of Generalities.

*10. What we term the Perception of an individual, as a given tree, is not simply a sense impression of the moment, it is an aggregation of many generalized impressions.

When we look at a tree, we are affected by a great number of different influences—colours, shape, size, &c. Now, every distinguishable impression recalls the previous stamps of the same, by Agreement or Similarity; and the idea of the tree is not an original sense presentation, but a compound of this with old presentations. Every feature of the tree suggests a classification upon that point; the green and brown colours are felt only as the collective impressions of those shades of colour.

In our minds, therefore, the Concrete and the Abstract are inextricably blended. Of a pure concrete, not also resolved into classifications or abstractions, we have no experience. Our knowledge proceeds in both ways at once; individuals giving generals and generals re-acting upon individuals. If there was one concrete thing in the world, having no property in common with any other known concrete thing, we might, by gazing upon that, and comparing it with itself, possess an idea of a concrete individuality, where no generality was implicated; but such a concrete would be very different from any concrete known to us. We are not in the position to imagine such an idea.

11. The speciality of a concrete Individual is that it is a definite aggregate not confounded with other individuals.

The number of general properties pointing to the individual must be such as to give it a definite or special character, instead of leaving it indefinite or common. The tree that I now look at, is individualized by a concurrence of properties never realized before; or if not by such concurrence itself, by its surroundings, and all the circumstances of time and place, accompanying its perception. A shilling is individualized by its adjuncts of place and time.

.12. The distinction between *Presentation* and *Representation*, is the distinction between a definite conflux of generalities, and an indefinite conflux.

A shilling in the hand is a Presentation. A shilling as a general coin of the realm is Representative; it is common to

many places and times and circumstances, and not bound down to one definite situation and one definite moment.

13. The names of Individuals usually correspond to their character as a conflux of generals.

In a few instances, we have names that bear no reference to generalities, as when a certain individual man is named—Cæsar. These are proper, or meaningless names; the bare symbols for separating the thing from other things. But in the vast majority of instances, the name follows the manner of conceiving the thing—that is, by specifying the concurring generalities. A large gothic building; a stout man of forty; a cubical crystal, with a certain hardness and specific gravity, found in a certain formation:—are examples of designations in strict accordance with the ideas of the things.

Philology confirms this. The primitive names of such concrete objects as sun, moon, father, mother, have all a generalized meaning; 'moon' is the measurer, 'father' is the feeder, and so on. There seems to be no possibility of conceiving individuals without classifying and generalizing at the same time. and the one name means both an individual and a general.

The intellectual function of Agreement, or Similarity, as the basis of Reasoning.

14. Reasoning, in every form, supposes the operation of Similarity—the assimilating of one thing to some other thing.

The most general type of Reasoning is to infer from one particular fact to another particular fact of the same kind; the likeness being both the means of suggestion, and the justification of the transfer of properties. We throw a stone into a pool; it makes a splashing noise, sinks to the bottom, and diffuses a series of waves from the point where it fell. We infer or reason, or presume, that another stone thrown into the same pool will be followed by the same series of effects; and we may extend the inference to another pool, or to any mass of liquid. This is to infer, to reason, to transcend our actual experience, to make an affirmation respecting the unknown. Now, the mind is prompted by the likeness of the cases to take this step in advance, to anticipate what is to happen. One would not infer that a handful of dried leaves

would produce all the consequences of throwing the stone; we never expect either through our instinctive belief, or through our experience of the world, that the same effects will arise under different circumstances.

This mode of Reasoning is in constant use, and extends to the animal intelligence. An animal accustomed to find a shelter under a bush, reasons from one bush to another bush, being moved solely by the resemblance of the second to the first. A dog is deterred by the menacing movement of a strange person wielding a strange stick: the partial resemblance to former experiences is enough to rouse its fears.

A second mode of Reasoning is when by the help of general language, we infer from one or a few cases, to all cases of the kind: as when we conclude, after a certain number of trials, that all stones sink in water, that all matter of vegetable origin is combustible, that all animals are generated from other animals. This is *Induction*, in the more technical sense—the inferring not from particulars to other particulars, but from particulars to universals. The mental process is still *Similarity*, or the process whereby one thing suggests other resembling things. It is by similarity that we assemble in the mind all kindred facts that have ever come under our knowledge; we then are able to compare the points of agreement, with a view to an accurate general statement, in other words, an *Inductive* proposition.

The third kind of Reasoning, called *Deductive*, is also based on the tracing of resemblance. When we infer that, because all stones sink in water, a certain body will sink (which is *Deduction*), it is because that body resembles the rest, or has the points of community indicated by the general word 'stone.' When we have mastered a general principle, it is by similarity that we discover cases to apply it to, and so extend our knowledge deductively.

Origin of our Knowledge in Experience.

15. Our knowledge of the world, both of matter and of mind, is the result of our conscious Experience.

As regards the Material, outer, or *object world*, we gain our knowledge through the ordinary Senses, coupled with our Movements, under the three laws of our INTELLIGENCE—viz., Difference, Agreement, and Retentiveness. We see, hear, touch, taste, smell; we have our active energies aroused by things resisting, by movements, and by things extended; we

discriminate and identify impressions ; we acquire permanent recollections, and associate things presented in combination ; and, by all these processes (exemplified at full length in *Mental Science*, or *Psychology*) we lay up our stock of imagery, ideas, or thoughts, of the world of sensible experience.

As regards the *Mind*, or the knowledge of our inner life the senses do not avail us. We are directly and immediately conscious of our feelings, thoughts, and volitions, and acquire a store of permanent recollections of these also. We remember our different pleasures and pains, and the order of their occurrence ; we learn not merely things, but our ideas of things, and the laws of the rise and succession of these ideas. Thus, it is a fact of our mental or subjective life, that we easily recall to mind whatever strongly engaged our attention in the reality.

16. It has been alleged that some parts of our knowledge, instead of being the result of experience, like the greater portion, are *intuitive* or inherent to the mind, apart from the operation of the senses upon actual things, or the particular phenomena of the subjective consciousness.

At different stages in the progress of Philosophy, there have been given different lists of intuitive, or *à priori* elements of knowledge. At the present day the controversy turns chiefly on these four notions—Time, Space, Substance, Cause.

It is maintained that there is in these notions something that experience could not give ; so that some different origin must be sought for them.

On the other side, the supporters of the Experience theory hold that the Moving energies, with the Senses and Self-Consciousness, aided by the intellectual functions, can account for all these notions.

For example, TIME is an abstraction : and, like all other abstractions, is, properly speaking, a certain mode of likeness among individual things or feelings of the mind. All our experiences, whether object or subject, are regarded by us as more or less *enduring* ; the attribute of Time is the assimilation or classification of enduring states, as enduring. Apart from these actual experiences of differences and agreements of enduring things, there can be no such thing as Time, unless on the exploded doctrine of Realism, nor any self-subsisting *notion* of Time, unless on the erroneous theory of Conceptualism. In the absence of objects and states continuing or enduring, an intuition of Time is a self-contradiction ; in the

presence of such experiences of enduring things, discriminated and compared on the point of endurance, we cannot but have an idea of Time.

Next as to SPACE, or Extension, the fact common to all Matter, and not pertaining to mind. Extension belongs both to solid matter, and to the intervals between the masses of solid matter, which intervals are measured by the same sensibilities, namely, the muscular feelings of motion, supported by the passive sensations.

The *a priori* philosophers allege that Space comes from no experience, but is already inherent in the mind before anything is perceived; being the condition of our perceiving things external.

In opposition to this view, it is contended that Space in the abstract is merely the community or similarity of extended bodies, and of the intervals between them, commonly called empty space. We compare all those things on this particular point of agreement; we occasionally think of them under this comparison; and in so doing we are thinking of Space. This is the only view compatible with Nominalism. An innate form of Space is a species of Conceptualism.

The pure intuition of Space is said to be the source of our knowledge and belief of the Axioms of Geometry, this being held to have a character that no experience can explain. In the case of these Axioms, the *a priori* revelation takes the form of Principles, and not of mere Notions; but the fact is the same, although differently viewed. 'That two straight lines cannot enclose a space;' 'that things equal to the same thing are equal to one another:' are held by those that contend for an intuition of space, to be intuitive.

The idea of CAUSE is included among the alleged intuitions. It may be expressed either as a mere Notion or as a Principle, namely, 'that every effect must have a cause.' An equivalent proposition is, 'that nature is uniform or that what has been will be.' The contention is, that while, by experience, we might become aware that particular effects follow the law of Cause, or of Uniformity, we could not from experience know that every effect has and must have a cause, that what has been will always be.

The idea of SUBSTANCE means that, underlying all the phenomena or appearances of Matter and of Mind, there is an unknown or unknowable substratum, called Substance, Noumenon, Permanent Existence. This idea we cannot possibly obtain from experience; the very statement of it, shows

that it passes beyond experience ; yet some philosophers contend that we are obliged to assume and believe in it. •

As applied to Mind, Substance is another name for Personal Identity, or the supposed continuity of each one's mental existence—the canvass that receives and holds together all the feelings, thoughts, volitions, that make up the stream of our conscious life.

According to the counter doctrine on this head, the notion of Substance is fictitious, incompetent, and unnecessary. The real meaning of Substance, as applied to matter, is the point of community of *all material bodies*, the most highly generalized fact respecting them ; otherwise expressed by Resistance, Inertia, Momentum, the Mechanical property of matter. The meaning of Substance as applied to Mind is the most highly generalized property or properties of mind—the facts wherein all minds agree on comparison, and which caused them to receive the common designation Mind, as opposed to not-mind, or matter. These generalized points of community are Feeling, Volition, and Intellect, the three facts attaching in various degrees to whatever is accounted Mind.

The nature of Belief as applied to the controversy respecting the origin of Knowledge.

17. There is a natural tendency to believe much more than we have any experience of.

The primitive disposition of the mind as regards belief is to suppose that whatever is will continue, that what exists here and now, exists everywhere and at all times. This in-born credulity is checked and abridged by our experience ; we soon discover that we have been assuming too much ; and by degrees we abate our confidence and adapt our views to the reality of things.

The following are common examples of the tendency. Before experience, we believe that as we feel now, we shall always feel ; that other people feel as we do ; that what happens to us happens to all ; that whatever any one tells us is true. By the natural impetuosity of the mind, we form these assurances ; experience did not create them, but rather moderates and checks them.

That we should treat any partial experience as universal, being thus a consequence of blind instinctive forwardness, is no proof of what really happens in nature. As we are so liable to extend our assertions beyond the facts, we should be par-

ticularly on our guard against universal declarations. This is one of the weaknesses of human nature, and a leading source of fallacy and error.

To make the application to the particular case of causation. We are very ready to fall into statements as to the universality of cause and effect; but so we do with many other things, where we find ourselves utterly wrong. The real evidence of the Law of Causation must be something different from our being disposed to believe it

Nothing can be affirmed as true, except on the warrant of experience.

18. As the natural disposition to believe carries us into falsehood, we must, notwithstanding our instincts, cling to experience as the only standard of truth.

This inevitably follows from the nature and sources of Belief. Even the supporters of innate principles, at the present day, admit that these principles cannot arise except along with the actual things; a qualification that subjects the innate notions as completely to the measure of experience, as if there was nothing innate about them. Our intuition of Cause is supposed to show itself only when we have observed a number of examples of cause and effect; it is, therefore, involved and implicated with our experience to such a degree as to be deprived of an independent standing. There is no means of discovering what the intuitions would dictate of themselves. For all purposes of logical certainty, therefore, they must be put out of account; regard must be had solely to observation, and experience.

Our Knowledge is Limited by our Sensibilities.

19. We are able to know what things affect our various sensibilities, or what may be compounded of these; and our knowledge extends no farther.

We have a certain number of sensibilities, namely, in the Senses (Passive), and in the Muscles (Active); and when any of these is affected we have knowledge or experience; we know sights, sounds, touches, tastes, smells, and various organic affections, we know resistance and movement. We know various emotional states, love, anger, fear, &c. We have many experiences from the discrimination and

the agreement of our various states. In these, we have our alphabet of the knowable. We can then combine a number of primitive feelings into a constructive aggregate, as when we attain to the idea of an orange, or of a man, or of the entire globe. But we cannot by any effort pass out of the compass of these primitive sensibilities. Supposing the universe to contain powers and properties that do not impress one or other of our senses, as at present constituted, we can never by any means be made cognisant of such properties.

On this ground the notion of a Substance distinct from all attributes is a thing unknowable. We can know body by its sensible properties, and mind by our conscious feelings, thoughts, and volitions; and we can know nothing beyond.

FIRST PRINCIPLES OF LOGIC.

20. In Logic, there are certain general principles, constituting it a science properly so called, and lying at the foundation of its practical rules and methods.

These principles are variously expressed. They are termed Laws of Thought, and fundamental Axioms of Reasoning. From embracing these highest of all generalities, which penetrate into every science, and from laying down rules on scientific method, Logic has been designated '*scientia scientiarum*'—the science that comprehends all sciences.

The First Principles may be arranged thus:—

- I. The Principle of CONSISTENCY, or Necessary Truth.
- II. The Principles of DEDUCTION.
- III. The Principle of INDUCTION.

I.—Principle of Consistency—Necessary Truth.

21. It is a fundamental requisite of reasoning, as well as of communication by speech, that what is affirmed in one form of words shall be affirmed in another.

Language often contains equivalent expressions for the same fact. There are synonymous names as 'round,' 'circular;' a round thing is the same as a circular thing. 'Matter is heavy,' 'matter gravitates' are the same fact in different words; if the one is true, so is the other, by virtue of mere consistency. Again, there are forms that enable us to affirm many separate facts in one sweeping statement; instead of affirming in detail, Mercury moves in an ellipse, Venus moves in an ellipse, &c.,

we can put forth the one condensed affirmation—all the planets have elliptic orbits. Having advanced this general statement, we are required by consistency to maintain each separate particular, the orbit of Saturn is elliptical, and so on.

It is obvious that without this consistency, there could be no intelligent communication between one human being and another. Unless the affirmer adheres to his affirmation, however he may vary the language, no one can divine what he means; there is no possibility of discussion or reasoning.

To these self-consistent, although variously worded, affirmations is applied the description 'Necessary Truth.' 'All matter is heavy, therefore any one piece of matter is heavy' is called a *necessary inference*. A more exact designation would be an *equivalent, implicated, or self-consistent* assertion.

There is a vital contrast between passing from one form to another form of expressing the same fact, and passing from one fact to another distinct fact. When we say—because both A and B are mortal, therefore, A is mortal—we merely repeat ourselves; when we say, because A is mortal, therefore B is mortal—we make the affirmation of one fact, the ground of an affirmation of a different fact. In order to the one leap, we need only to know the meaning of language; in order to the other, we must consult the facts of the world.

The supposition has been advanced that truths of implication or consistency, inappropriately called 'Necessary,' are drawn out from their equivalent statements by a peculiar innate power of the mind, distinct from the powers of observing the order of nature; that without a special instinct they could not be evolved, nor reposed in with the absolute credence that we give to them. There are no sufficient grounds for the supposition. We should be disposed to consistency of statement, without any special instinct. The impossibility of carrying on intercourse by language, on any other footing, compels us to be consistent in our statements; at least up to a certain point, for we are not always so. There is no instinct needed but the broad instinct of self-preservation; were it not for this we should probably care very little about observing the conditions of necessary truth. If we could go on as well by maintaining an opinion in one form of words, while denying it in another, there appears to be nothing in our mental constitution that would secure us against contradicting ourselves. Our faculties as laid down by those philosophers that derive all our knowledge from experience alone, taken together with our practical necessities, seem quite sufficient to make us ad-

here to our statements under all variety of forms and expressions.*

22. There are certain maxims of Consistency known by the title 'Laws of Thought'; they are the principles of *Identity*, *Contradiction*, and *Excluded Middle*.

The principle of *Identity* is given in the form "A is A"; a thing is what it is; man is man. According to Plato, "The Idea is equal to itself."

Properly speaking this is not the case contemplated under the principle of Consistency; it is not the same fact in other language, but the same fact in the same language. That the same meaning expressed by the same word or words, is the same, would appear to be an utter superfluity of affirmation; what we want to be guarded against is mistaking the same fact in a different form of language.

This obvious criticism is evaded by giving the law an interpretation that supposes difference in the statement. The meaning is said to be that the thing A, although differently worded, is still A; which is merely an awkward way of stating the general maxim of Consistency. If A equals, or includes, a, b, c, d, &c., then we may say, in slightly different words, A is equal to the whole series of what it includes; a whole is the sum of its parts; a complex attribute is the aggregate of the component attributes.

The Principle of *Contradiction*. 'The same thing cannot be A and not-A'; this room cannot be both hot and not-hot, that is, cold. Consistency requires that when we affirm a definite fact, we do not at the same time deny it; having made an assertion, we are to abide by that. The principle may be carried one step farther. By the law of Relativity, every thing that can be thought of, every affirmation that can be made, has an opposite or counter notion or affirmation; to the thing that we call a 'straight' line, there corresponds a negative or opposite called a 'bent' or crooked line. Now thorough-going consistency requires that when we affirm a certain thing to be

* Only some of the *a priori* philosophers, as Leibnitz, contend for the existence of an intuitive faculty in order to apprehend these judgments of mere consistency. Kant, and others after him, confine the characteristics of necessity, and of intuitive origin, to certain *synthetic* judgments, where the two things given are distinct, and not mutually implicated facts. It was the peculiarity of Kant to maintain that there are such synthetic judgments *a priori* transcending our actual experience: he instanced, in particular, the proposition that 'two straight lines cannot enclose a space.'

a straight line, we must be prepared also to deny that it is a bent line; when we call this man wise, we must also deny that he is foolish. This is an equivalent form that plays a great part in Logic. Viewed thus, the Law of Contradiction has a pregnant meaning, which can hardly be said of the Law of Identity.

The Principle of *Excluded Middle*. 'A thing must either be or not be;' 'of contradictories one must be true, and the other false.'

This law grew out of the distinction of propositions into those of total, or universal, and those of partial or particular quantity—all men and some men.' When a proposition of universal quantity is opposed by one of particular quantity, the opposition is not thorough-going; there is not a perfect and entire contrariety. Perfect contrariety is between, 'all men are mortal' and 'no men are mortal;' partial or incomplete contrariety is 'all men are mortal,' 'some men are not mortal;' and 'no men are mortal,' 'some men are mortal.' Between this last species of opposition, there is no middle affirmation; if one is not true, if it is not true that all men are mortal, then it must be true that some men are not mortal; we have no third alternative. But in the thorough-going contrariety—'all diamonds are precious,' 'no diamonds are precious,' there is a middle ground of compromise; the fact may be that some diamonds are precious and some not. Thus, the Law of Excluded Middle is an incident of partial or incomplete contrariety. It was enunciated by Aristotle as following from the classification of propositions according to quantity. It is too much honoured by the dignity of a primary law of thought.

The Principle of Consistency, inadequately rendered by these Laws of Thought, may be assigned as the basis of the logical department entitled 'Immediate Inference' (as opposed to Mediate Inference or Syllogism), 'Inferences improperly so called,' 'Equivalent Propositional Forms.' Whatever be the general designation, the details are fully agreed upon; the doctrine of the Conversion of Propositions is one of the leading topics.

• *First Principles of Deduction.*

23. In Deduction, there is the application of a general proposition to a particular case coming under it.

The following is a deduction:—'All arsenic is poison; now this substance is arsenic; therefore, this substance is poison.' This is something more than consistency, implication, or

equivalence of phraseology. There would be equivalence of affirmation in saying 'all arsenic is poison; therefore, *some* arsenic is poison.' In the present case, however, we have another step to take; we need a second and distinct assertion, '*this substance is arsenic*,' before we can conclude, '*this substance is poison*. Instead of deriving an affirmation from a prior affirmation, by change of language, we derive an affirmation from *two* prior affirmations; and these have to be related one to another in a proper form, in order that we may draw the conclusion.

This process is called *Mediate Inference*; there being an intermediate link or stepping-stone between the primary proposition and the conclusion. We cannot, by mere Consistency, resolve 'All arsenic is poison' into 'the substance in this bottle is poison'; 'no matter is destructible,' into 'no ether is destructible'; there is in both cases a missing link. Until we show that the substance in the bottle is arsenic, and that ether is matter, we cannot draw the special conclusions above given.

24. The Axiom, or First Principle, at the basis of Deduction, is expressed in a variety of forms, which are reducible substantially to two:—

(1.) Whatever is true of a whole class is true of what can be brought under the class.

(2.) Things co-existing with the same thing co-exist with one another.

There are corresponding forms for negative reasoning.

The first form is the one suitable to the exposition of the syllogism. It sets forth the deductive type of reasoning, as consisting of a general principle brought to bear upon a case or cases, found to come under it.

The second form can be shown to be equivalent to the first. It has the advantage of making prominent the *mediate* character of deductive inference, so as to contrast it with immediate inference, or mere identical propositions under the Law of Consistency. Two things not known in themselves to co-exist, are shown to co-exist by each co-existing with some third thing. Mere consistency will not include this case. The principle is admitted as soon as it is understood; but solely because each one's experience bears it out.

The obverse forms, for negative reasoning, are—(1) What is denied of a whole class is denied of whatever can be

brought under the class; (2) One thing co-existing with a second thing, with which second thing a third thing does not co-exist, is not co-existent with that third thing.

25. The Axioms of Deduction suppose the Uniformity of Nature.

This is obvious, if the axioms are based on experience. We have observed, in a large number of instances, that things coinciding with the same thing coincide with one another; but we have not observed it in all instances; we have not observed it in what took place before we were born, in what is beyond our reach, or in what is still to happen. Yet, from the cases we have observed, we do not hesitate to extend the principle to the unobserved cases. We thus assume that 'nature is uniform;' that what we find to-day, all circumstances being the same, we shall find to-morrow.

Again, we may deny that the axioms are experimental, and call them intuitive. The case is not altered. The intuition still supposes nature's uniformity; the thing intuitively conceived and believed is not true, unless nature be uniform. Thus, on either supposition as to our knowledge of the Logical (and Mathematical) Axioms, the truth, still deeper, and more comprehensive, is that nature is uniform. The so-called axioms, therefore, are not ultimate principles; they are only secondary, proximate, or derivative; they proceed from a stem bearing other branches besides them. If they are true, more is true. The wider principle will next be stated, for the sake of its other consequences.

First Principle of Induction.

26. When we infer from a fact known, to another unknown, we make a *real* inference, for which there must be some guarantee.

The sole guarantee is the Uniformity of Nature.

Putting a piece of wood into the fire and seeing it consumed, we infer that another piece will be consumed in like manner. This is to take for granted that what has happened will, in the same circumstances, happen again; in other words, that Nature is Uniform.

The Uniformities of Nature fall under (1) Uniformities of Co-existence, and (2) Uniformity of Succession. It is a uniformity of Co-existence that 'inert matter gravitates,' that the distinctive property of matter called 'Inertness' is asso-

ciated, through all nature at all times, with the property of weight or Gravitation.

The evidence for Uniformities of Co-existence is special observation of each separate uniformity. From seeing two things coupled together in a few instances, we cannot presume that they are always coupled together; we must observe the coupling in various times, places, and circumstances. If, after a sufficient search, we find no single contradictory instance, we affirm the union to prevail through all nature.

27. In Uniformities of Succession, there has been discovered a *law* of Uniformity that shortens the labour of enquiry in this department. It is called the Law of Cause and Effect, or Causation. We may express it thus:—

‘Every event is uniformly preceded by some other event.’
 ‘To every event there is some antecedent, which happening, it will happen.’

To say that ‘Every effect must have a cause,’ is begging the question; the word cause implies an effect, and the word effect implies a cause. The correct mode of expression is, ‘To every event there corresponds a prior event, which happening, it will happen; and which failing, it will not happen.’ ‘The antecedent may be, and often is, a whole assemblage of circumstances; as in the case of Health, an effect depending on many conditions.

Since there are effects produced by a plurality of Causes, the principle of Uniformity is limited and qualified by that circumstance. Thus, Death may be caused by starvation, by a violent blow, by poison, &c. It is therefore proper to say that given any of these conditions in sufficient amount, death will follow; but the occurrence of death does not prove that there has been starvation; it proves only that *one* of the producing agencies has been present. In the Inductive enquiry into nature, *all* the causes that may produce each effect are sought out.

From the Law of Causation, we deduce consequences such as these:—‘If the cause be absent, the effect will be absent’—‘*cessante causa, cessat et effectus*,’ ‘If the cause be present the effect will be present,’ ‘Whatever agent cannot be removed without the cessation of the effect, must be the cause or part of the cause,’ ‘Whatever agent can be removed without the cessation of the effect is not the cause,’ ‘The cause and effect vary proportionately’

These various aspects or implications of the Law of Causation are the maxims serving to eliminate and to prove cause and effect in the phenomena of nature.

28. The Law of Uniform Causation appears in a form still more pregnant with consequences, namely, the Law of the Persistence, Conservation, Correlation, or Equivalence of Force.

This is a generalization only recently effected.

Galileo and Newton may be considered as having established the Law of the Persistence or Conservation of *Mechanical Force*, that is, force applied to matter in masses. If one ball strikes another and puts it in motion, the force imparted to the second is exactly what is lost to the first.

Lavoisier established the persistence of ponderable matter, showing that no atom of matter could be destroyed, and none created. In burning and in evaporation, the particles merely change their positions; they do not abandon their material properties of inertia and gravity.

In the present day, evidence has been obtained to show that *other* forces besides mechanical force, namely, Heat, Chemical Force, Electricity, Nerve Force, have the same numerical persistence; they can neither be created nor destroyed; They can, however, be mutually converted, at a definite rate. Heat can give birth to Mechanical Force; Chemical Force can evolve Heat; Electricity is convertible into all the other modes. In this conversion, nothing is lost, and nothing is created; when heat becomes a mechanical prime mover in the steam engine, it disappears as heat. When mechanical force is seemingly destroyed, as when a cannon ball spends itself on an unyielding mass of stone, the whole momentum of the ball is transformed into heat; at the place of encounter, both the ball and the stone are raised in temperature, exactly in proportion to the momentum arrested.

This great law of the quantitative persistence of Force, or Momentum, deserves an eminent place in the Inductive Logic. It encompasses and pervades all the natural sciences, each one of which is but a partial development of it.

NATURE AND CLASSIFICATION OF KNOWLEDGE.

29. Knowledge is made up of affirmations respecting the order of the world. These affirmations are the subject of Belief, of which the ultimate criterion is Action.

Twice two is four; the sun rises and sets; unsupported bodies fall to the ground; heat causes water to boil; animal bodies are nourished by food and air; harmony is agreeable to the mind:—are affirmations, or Knowledge, respecting the universe. We believe them, and show our belief by acting on them. When we desire water to boil, we apply heat; which is our belief of the affirmation.

30. The first requisite of Knowledge is that it shall be *true*.

An Affirmation is true when, on actual trial, it corresponds to the fact. This is the direct proof. Indirectly, we may test the truth of affirmations by comparing one with another. Wherever there is contradiction, there must be falsehood.

31. Knowledge is either Particular or General.

An Affirmation respecting a certain individual thing, as 'this house is stable,' 'Cæsar was brave,' 'a certain patient will not recover'—is a particular or individual affirmation; it is limited to one subject. An affirmation respecting a whole class or species of things—as 'an erection is stable when the line of the centre of gravity falls within the base'; 'all great generals are brave'; 'the stiffening of the limbs is a sign of death';—are general affirmations; they extend to instances beyond number.

32. Owing to the frequent recurrence of the same things and the same processes, we can attain to numerous generalities.

If every individual thing in nature were throughout unique, resembling no other thing, each would need a law to itself. If, instead of a common substance 'water' in all seas, rivers, and fountains, there were a thousand different substances, we should have to multiply affirmations accordingly. If, instead of the sixty-three elementary bodies known to us at present, the globe were made up of six thousand elements with their compounds, there would be a great increase in the bulk of our knowledge. If instead of sixty-three, there had been six, we should have been able to comprehend all physical knowledge in comparatively few affirmations.

33. It is desirable to attain knowledge in the highest possible degree of *generality*.

The reason is obvious. A general affirmation is a great many particular affirmations in one. It is a vast economy of the human understanding. A general law places us at a commanding height, where, by one glance, we can survey a wide array of facts. The law of Gravity, the law of the Persistence of Force, the law of Definite Proportions in Chemistry, the law of Relativity in Mind,—severally comprehend thousands of individual affirmations.

34. The perfect form of knowledge is SCIENCE.

The peculiarities of Science are these :—

I. It employs special means and appliances to render knowledge *true*.

The uninstructed man is apt to make affirmations without taking the trouble to test them. The scientific man, on the other hand, not only avails himself of the common means of proof, but employs an express machinery for testing all the knowledge in his own department. This machinery is to a certain extent common to all knowledge, and all science; and to a certain extent, it is special to each science. The common machinery is embraced in Logic.

35. II. Knowledge, in the form of Science, is made as *general* as possible.

Science does not refuse individual facts, provided they are true; on the contrary, it collects as many such facts as possible. But considering the enormous sweep and vantage ground of generalized facts, science pushes the generalizing process to the utmost limits. A few isolated facts carefully ascertained to be true, would be valuable in themselves, but they would not constitute a science.

36. III. A Science embraces a *distinct department* of the world, or groups together facts and generalities that are of a kindred sort.

It appears, on investigation, that the operations of the world are different in their nature, and need to be differently studied. The forces that maintain the motions of the heavenly bodies, are different from combustion, magnetism, or vegetable and animal growth. The functions of the mind scarcely resemble anything else. Hence the affirmations or truths respecting the world fall into distinct departments; and there is an evident propriety in observing the distinction, and in classing kindred facts together. So class together facts about the

planets, and facts about the human mind, could only perplex the understanding.

37. IV. A Science has a certain *order* or arrangement of topics, suitable to its ends in gathering, in verifying, and in communicating knowledge.

Besides bringing together the facts and generalities relative to each division of phenomena, a science must present its materials in a fitting arrangement.

This arrangement varies in the different sciences. Still, in all of them, attention must be given to the following points—

(1) To proceed from the more easily, to the less easily known. If any fact or generality depends upon or presupposes another, that other should be stated first in order.

(2) Whatever is requisite for proving any doctrine should precede what is to be proved. In concatenated or deductive sciences, like geometry, each affirmation depends upon some that go before; and the evolution is thus methodical and systematic.

(3) The meanings of all terms should be distinctly given before they are made use of. It is usual to commence with the definitions of leading terms.

38. The classification of the Sciences is in accordance with the foregoing views. In the first place, it follows the division of nature into departments, and in the second place, it follows the order of relative simplicity and of *mutual* dependence in those departments.

If each different process of nature were entirely separate from the others, there would be no special *order* of the sciences. But the distinct powers—gravity, heat, animal growth, mind, &c., are to a great degree intermingled in their workings. Moreover, all phenomena whatever are subject to laws of Quantity, and these can be studied apart from any one class of things; hence, such laws are a preparation for all the departments. Nor is this the only way that one science paves the way for another. Accordingly, there is, among the several sciences, an order of dependence that, to a certain degree, determines their succession to the learner, and their gradual evolution under the hands of scientific enquirers.

39. The Sciences are either Abstract or Concrete.

The Science of Mathematics, which treats of *quantity*, without referring to any particular kind of quantity, as length,

weight, heat, &c., is called an Abstract Science. With one exception, it is the most abstract of all the Sciences; the properties treated of are the most general of all properties; and they are discussed in the highest degree of separation from concurring attributes.

On the other hand, Zoology, which classifies and describes one great department of actual or concrete things—the whole Animal Kingdom—is a Concrete Science.

The science that, in point of abstractness, rivals Mathematics is Logic itself. The First Principles of Logic, as above laid down, including the law of Consistency, the law of Deduction, the law of Uniformity, are paramount over every science; they are wider than even the laws of quantity.

Next to quantity, the most general attribute of natural things is *motion*. All material bodies may pass into motion—motion in mass (molar movement) or motion in molecule (molecular movement) or both. Now the laws of motion may be laid down without reference to any particular objects. Hence there may be an abstract science of Motion, for which the name might be Abstract, Theoretical, or Rational Mechanics; the designation now accepted is ‘Kinematics.’ The principles of motion, as applied to actual bodies—solids, liquids, and gases—constitute the departments of Concrete Mechanics, which have appropriate names.

The Abstract is also the *simple*, the concrete is usually the *complex*. When what is true of the Abstract is not also true of the concrete, the reason is an incident and not a necessity. What is true in the Abstract really means truth in the concrete; the abstract is merely a name for the concrete under agreement. A law true in the abstract would be a contradiction, if it were not true in the concrete also. But in the concrete, there may be counteracting forces, so that the real point is to contrast a power working *alone* with a power working in *company*. The abstract law of motion—the persistence of a body in its present state, fails in the concrete, because of friction, or of opposing obstacles; the tendency to persist is compounded with other influences, and we have to calculate the result of the composition. Self-interest working alone would have certain consequences; as an element of a compound, it is no longer accountable for the whole effect.

The Abstract Sciences properly precede the corresponding Concrete Sciences.

40. For the purposes of the present day, the Sciences may be classified as follows:—I. Logic, II. Mathematics, III. Mechanics or Mechanical Physics, IV. Molecular Phy-

sics, V. Chemistry, VI. Biology, VII. Psychology. In every one of these, there is a distinct department of phenomena; taken together, they comprehend all known phenomena; and the order indicated is the order from simple to complex, and from independent to dependent, marking the order of study and of evolution.

I. LOGIC embraces, as has been seen, the most fundamental and universal of all principles—Consistency, Deduction, and Uniformity. It reposes upon nothing more fundamental than itself, and it gives foundation to all the other sciences. There can be no science without assuming all the data of Logic, whether avowedly or not.

II. MATHEMATICS is the abstract science of Quantity, and the laws of Quantity, in every possible combination.

III. MECHANICS, or Mechanical Physics, or Mechanical Philosophy, is the science of Motion, as regards bodies in *mass*, and of Force, which is the momentum of moving masses. There is an abstract or theoretical department (Kinematics), comprising all the laws of the Equilibrium, and of the Movements, of matter in mass, without reference to any special class of things. The *Concrete* applications of these laws embrace Astronomy, or the Celestial Motions, the kindred subject of Falling Bodies on the Earth, Statics, Hydrostatics, Dynamics, Hydrodynamics, Acoustics.

IV. MOLECULAR PHYSICS refers to the *molecular* movements and arrangements of material bodies. It comprises the Molecular Cohesions and Adhesions, as operative in the structure of Solids, Liquids, and Gases; Heat; Light; Electricity.

V. CHEMISTRY is a continuation of Molecular Physics, having more especial reference to the Combinations and Decompositions, named chemical, and characterised by great accompanying changes of properties.

The branch of Science, long known as Natural Philosophy, comprises both Mechanical Physics and Molecular Physics, but excludes Chemistry. An equally, if not more, suitable arrangement would be to treat Chemistry as a part of Molecular Physics; into which it shades by imperceptible gradation. In point of fact, Chemical action is inseparably implicated with Heat and with Electricity, although these subjects can be, in exposition, detached from Chemistry.

Mechanical Physics and Molecular Physics, taken together, exhaust all the fundamental aspects of the great doctrine of the Persistence, Conservation, or Correlation of Force.

VI. BIOLOGY enters upon an entirely new field of phenomena, the phenomena of Life, or of Living Bodies, involving an organised structure, with perpetual evolution and reproduction. This science is posterior to the foregoing, inasmuch as living bodies come under all the laws of Mechanical and of Molecular Physics, in addition to their own specific laws as living bodies.

Biology is divided into Vegetable and Animal Biology; the one exhausting the structure, classification, and description of Plants, the other referring to Animals. Botany, Zoology, Human Anatomy and Physiology, are the *concrete* departments of Biology, and its leading divisions for study. There can scarcely be such a science as Abstract Biology; the laws of life cannot be given in separation from living vegetables and animals. The nearest approach to a division into Abstract and Concrete, is the distinction between Physiology—Vegetable and Animal—on the one hand, and the classification and detailed description of Plants and of Animals on the other.

VII. PSYCHOLOGY, or the Science of Mind, is a unique department of natural phenomena. Its terminal position in the order of the Sciences is owing to two circumstances. In the first place, it is a subject of great complication, aggravated by an especial amount of corrupting bias. Hence the student does well to come prepared with a scientific discipline, such as is best furnished in the previously enumerated sciences. Secondly, although the mind proper—the subjective consciousness—is a unique subject, yet a material organism is allied with it throughout, and therefore should be known as so allied. Now the material organism falls under the last part of Biology, namely Human Physiology.

These seven branches contain the laws of every known process in the world, whether of matter or of mind; and set forth those laws in the order suitable for studying and comprehending them to the greatest possible advantage. No phenomenon can be strange to any one thoroughly conversant with those subjects. Properly speaking, the laws of the phenomena might be comprehended under four heads:—Molar Mechanics, Molecular Mechanics (or Physics), Biology, Psychology. Logic and Mathematics are merely aids to the better comprehension of the actual things.

Astronomy was detached, by Auguste Comte, from its usual position under Mechanics, and made one of the primary departments. His reason was that it deals with the great fact Gravity—a distinct and specific phenomenon, unlike everything else,

and capable of being developed apart, merely with the aid of Mathematics and abstract Mechanics. Although the position thus given to the subject may be thought unnecessarily prominent, yet the reason contains an undoubted and highly illustrative fact. The gravitating action is peculiar and distinct; it operates in the celestial bodies uncomplicated with any other actions, giving Astronomy a character of remarkable simplicity as regards the forces at work.

41. The Concrete Departments include various additional subjects—as Meteorology, Mineralogy, Geology, Geography, —no one of which involves any operation but what is expounded in the Fundamental or Departmental Sciences.

In each of these branches, a certain group of *locally* allied phenomena is separated for special study. *Meteorology*, treats of the Atmosphere, all whose phenomena are regulated by the laws of Mechanical and of Molecular Physics. The same may be said of *Mineralogy*; there is no natural agent at work in the formation of minerals, but what is described in the fundamental departments last named. The special aim of the subject is to provide a systematic mode of classifying and describing mineral bodies, so that they may be recognised and understood. *Geology* involves Biology, in addition to Physics; its locality is the crust of the globe, so far as accessible. *Geography* is the science of the Earth's surface—and is, like the two foregoing, a descriptive science, but containing no new laws of phenomena.

Among Concrete Sciences related more particularly to mind, we may class the Science of *Society*, Politics, or Sociology, which applies the laws of Mind to human beings aggregated in Society. Another example is Philology, or the theory of Universal Language, together with the Classification of the Languages now or formerly spoken.

42. We have not yet exhausted the branches of knowledge designated Sciences. There remain the PRACTICAL SCIENCES.

The final end of all knowledge is Practice, or the guidance of conduct. There are numerous departments of practice, according to the needs of human beings; and every one of these reposes upon knowledge more or less accurate. Another name for practice is Art.

Now, according to the quality of the knowledge at command, Art may be empirical or it may be scientific. An empirical art proceeds solely upon the knowledge gained in the exercise of the art itself. All arts were empirical before science began; as for example, Agriculture, Navigation, and Metallurgy. There are still some empirical arts, as the greater part of Medicine.

Art becomes scientific, when science is brought to bear upon it. Navigation is a remarkable instance; being aided by Mathematics, Mechanics, Astronomy, Optics, and Meteorology. Engineering, Building, Machinery, Dyeing, and the range of Manufactures generally, are arts founded on Science, and may be called Scientific Arts, or Practical Sciences. Another group (connected more with mind), includes Ethics, Logic (in its practical aspect), Æsthetics, Rhetoric, Grammar, Education, Politics, Jurisprudence, Law, Political Economy.

Several of the subjects last named might be viewed either as Theoretical Concrete Sciences, or as Practical Sciences. This would depend upon whether they were constructed most upon the one type or upon the other. Thus, Politics might be arranged as a methodical body of political doctrines consecutively evolved from primary truths or data, like Mechanics, Chemistry, or Psychology. It might also be arranged with a predominant regard to the *political end*, and might take the form of a series of maxims or directions for the art of government, more or less supported by scientific doctrines and general reasonings. A similar remark applies to Political Economy, Jurisprudence, and Ethics.

43. In a Practical Science, the knowledge is selected and arranged purely with reference to the object in view. The definition of a Practical Science is its End.

This makes a great difference as respects choice of topics, between a Theoretical Science (Abstract or Concrete) and a Practical Science. In the first, the knowledge imparted pertains exclusively to one department of natural phenomena—Motion, Life, Mind, &c. In the second, the knowledge is selected from one or more theoretical sciences, and set forth in the order suited to the end in view. In a theoretical science we obtain, in the most succinct and intelligible shape, the entire body of existing information relating to one group of kindred phenomena; the knowledge being applicable to several arts, but not specially applied to any. In a practical

science, the information conveyed is kept in subservience to the purpose of the art.

That the definition of a Practical Science is its End, was a point greatly insisted upon in the Aristotelian treatises. Thus, in *Ethics*, we have to ascertain first the *telos*, the ethical end; on which turns the chief differences of opinion on the subject. Logic, in so far as being a theoretical science, is defined by its natural department; as a practical science, or an art (whether empirical or scientific), it must be defined by its end. (See also Appendix A, and Inductive Logic, Book III.)

DIFFERENT VIEWS OF THE DEFINITION OR PROVINCE OF LOGIC.

44. Logic has been termed (I.) the Art of Reasoning and (II.) the Art and Science of Reasoning.

The first is Aldrich's definition; the second is Whately's amendment. In both forms, there is an admission of the practical character of Logic; in the second form, the practice is said to be founded on Science; in other words, Logic is a Practical Science.

45. The term 'Reasoning' is insufficient, as being, first susceptible of more than one interpretation, and, secondly too narrow for the admitted scope of Logic.

Reasoning may mean Deduction solely, or it may mean Inference as a whole, which is Deduction together with Induction. In the narrower acceptation, Logic would be confined to Deductive Reasoning, or Syllogism; in the wider acceptation, it comprises Induction also. The narrower meaning has been the most usual in Logical treatises, but in scarcely any one is it consistently adhered to. Either under the title of Induction, or as Applied Logic, matters pertaining to Induction have been introduced, by Whately, Hamilton, Thomson, and others.

Again, taken in its widest sense, the term Reasoning is still too narrow. We always find, in books on Logic, subjects not comprised under the term Reasoning: as Classification, Definition, and Division; all which are amenable to rules, and may be performed well or ill. We apply the epithet 'logical' to a definition, as well as to an argument.

46. III. Logic has been described as 'the Science of the Laws of Thought'

This definition remedies the narrowness of the foregoing in respect of the use of the word Reasoning. 'Thought' is large enough to cover all the processes admitted into Logic. It, however, does more; it includes in its meaning all the intellectual processes, being co-extensive with intelligence itself. Memory and Imagination would be departments of thought. Consequently, the word has to be narrowed in its signification; to what is termed 'Discursive' or 'Elaborative' Thought, the faculties concerned in the scientific operation, or in the attainment of truth; which faculties may be summed up in the two—Abstraction and Reasoning. The power called Abstraction covers those portions of the field of Logic that Reasoning in its widest meaning does not cover.

Even with this limitation, the title 'Laws of Thought' is liable to other objections. In particular, it points, by an obvious interpretation, to *Psychology* rather than to Logic. The Laws of Thought, or of Thinking, would appear most naturally to indicate the laws of the rise and succession of our thoughts as explained in Mental Science; in other words, the laws of the Association of Ideas.

This difficulty can be met only by arbitrary interpretations of 'Laws of Thought.' By some, the phrase is qualified by the word 'Formal,' which, however, does not relieve the perplexity. Do the 'Laws of Thought' mean Thought as it is, or Thought as it ought to be? If 'Thought as it is,' then the subject is pure psychology; if 'Thought as it ought to be,' there must be supplied some principle for checking or controlling the spontaneous thinking of the mind, which principle is the all-important element of the case, and needs to be explicitly stated.

Hardly any amount of explanation will convert into a good Definition a phrase of such ambiguous and uncertain scope as the 'Laws of Thought.' When the proper limitations are supplied, there can be found some other phraseology more suitable to indicate what is intended. If the meaning is 'Thought as it ought to be'—Right or Corrected Thinking,—a standard must be assigned, which standard can be nothing but the standard of what is true, and false; 'the end of thought,' Hamilton remarks, is 'truth.'

47. IV. Logic is defined (Port Royal Logic) 'the Science of the operations of the understanding in the pursuit of truth.'

Here three things are implied. First, Logic is a department of practice, scientifically conducted, that is, a Practical Science.

Secondly, whereas every Practical Science, and every Art, whether scientific or not, must have an End, the end of the science of Logic is the attainment of Truth. Thirdly, the means employed in this pursuit is an enquiry into the operations of the human understanding.

The two first positions can hardly be controverted. Logic, no doubt, has a certain theoretic aspect, to be considered presently, but its chief aim must ever be practical. Had the subject not been wanted as an aid to the search of truth, it would never have been called into existence.

The third position—that the means in Logic consists in an enquiry into the operations of the Understanding—admits of one criticism: This may be *a* means, but is not necessarily the sole means.

48. The foregoing definition is modified by distinguishing between two kinds of truths:—namely those known *immediately*, intuitively, or by direct consciousness; and those known by the *mediation* of other truths

The distinction is fundamental and important. Facts of present consciousness, as—I am hungry, I hear a sound, I am pleased, I am speaking, — are amenable to no laws or rules; they are final and conclusive of themselves. We cannot escape from them, we cannot be more or less convinced of them by any method of procedure. They are the ultimate data of each one's knowledge.

The other class of truths, by far the most numerous, are known not by direct, immediate intuition, or consciousness, but by the medium of some other facts, themselves immediate. That the sun has risen is a mediate or indirect truth; what is immediate is the sensation of light, and from that immediate fact, we infer or believe the other fact, 'the sun is above the horizon.' That I feel cold is an immediate truth, that another person feels cold is a mediate inference; the immediate fact being certain sensations of sight, or of sound, with which I have learnt to connect the fact of feeling cold. All the feelings and thoughts of other beings are known to us in this way.

Everything that is transacted in our *absence* must be known mediately, if known at all. And as intuitive knowledge is confined to time present, all knowledge of the *past* and of the *future* is necessarily mediate.

Now, a mediate truth is properly an Inference. When a thing is known, not in itself, but by some second thing related

to it, the knowledge is mediate or inferred; and the immediate fact is the Proof or Evidence of the fact so inferred. The fact that the air is below 32 deg. Fahrenheit, is inferred from the visible phenomenon of falling snow; the snow is the medium of inference, the proof or the evidence that the air is cold; the melting of the snow would be the proof that the air is becoming warmer.

All such inferences suppose a sure link of connexion between different phenomena. If A is the evidence of B, A and B must be known as joined together in the nature of things. Now, in order to our assurance of such connecting links, certain processes have to be gone through—namely, Observation, Induction, or Deduction. In performing these processes, we are liable to commit mistakes; we need a number of precautions; and these precautions are the rules of Logic.

As regards Immediate Truths, no such precautions or rules are necessary. The chief mistake that we are liable to on their account (and the mistake is a frequent source of error) is the confounding of an immediate truth with an inferred truth. We are apt to say that we are immediately conscious of what we only infer. The most notable instance is our belief that we *see* distance; whereas, in fact (according to Berkeley and the majority of scientific men), we do but *infer* distance; our immediate consciousness is only of colour and of the tension and the movements of ocular muscles, which are *signs* of distance, but are not themselves the fact of distance.

Thus, while there are certain things, admitted by all to be matters of intuition, or immediate consciousness, such as our sensations and emotions in their primitive character; and certain other things equally admitted to be matters of inference, or mediate cognition, such as the feelings of other men, the facts of testimony, and the generalizations of science;—there is, as often happens, a middle ground, or margin, where intuition and inference are blended and confused, and where what is accounted intuition by one man may be called inference by another. This happens with some of the most celebrated questions. The existence of the Deity is reckoned by some to be an intuition, or an immediate revelation of consciousness, a judgment *a priori*; by others an inference from design, or a judgment *a posteriori*: while most commonly it is viewed as both the one and the other. Again, our perception of a material world is accounted an intuition by Reid and Hamilton; while others deny it to be intuitive in the sense intended. In fact, the controverted questions

relating to the Origin of our Knowledge all lie upon the doubtful margin of intuition and inference.

49. As Logic deals with truths of Inference solely, the definition (according to Mill, amending the foregoing definition), should be 'the science of the operations of the understanding that are subservient to the estimation of Evidence.'

The estimation of Evidence must unquestionably be accounted the main function of the Logician. It is his business to lay down the tests of true and false, with a view to the establishment of the true.

Whether the Logician should give suggestions as to *discovery*, or as to the modes of arriving at suggestions to be verified by the logical tests, is an open question. Mr. Mill does not expressly include this in his definition, but in the title of his work he couples with the 'Principles of Evidence' the 'Methods of Scientific Investigation.'

50. In the present work, Logic is viewed—

First, as a Theoretical Abstract Science.

Secondly, as the Practical Science of Proof or Evidence.

Thirdly, as a body of Method auxiliary to the search for Truth.

First. Logic, as we have seen, lays down the most fundamental laws of all affirmation, and deduces inferences from these laws, embodying them in suitable formulas. In this view, it is the parallel of Mathematics, being equally a theoretical science, although greatly inferior to Mathematics in the extent and variety of its developments and applications. The evolution of syllogistic forms may be regarded as a theorizing process; these forms being systematically deduced from the supreme laws, or axioms, of Deduction. From the Inductive law of Causation, in like manner, are deduced inferences, convertible into canons of inductive elimination.

From regarding Logic in this theoretical aspect, the older logicians distinguished *Logica docens*, the 'teaching', and speculative side, from *Logica utens*, the 'guiding' and practical side. In recent times, De Morgan and Boole may be considered as exemplifying the theoretical development, and as illustrating forcibly the parallelism between Logic and Mathematics—the abstract sciences by pre-eminence.

Secondly. Logic is the Practical Science of Proof or Evidence. The conclusions of Theoretical Logic are of value in

discriminating between truth and falsehood, between sufficient and insufficient evidence. This is the useful part of Syllogism, of Inductive Elimination, of the theory of Definition, and so on. The immense theoretical developments of De Morgan and Boole pass beyond the known applications of Logic in the present state of our knowledge; although, like the Conic Sections, which lay unused for two thousand years, these elaborate formulæ may one day be turned to practical account.

In the present work, the laws of Evidence are regarded in their widest compass, or as embracing alike Deduction and Induction. The main reasons are—that Induction is, properly speaking, the foundation of all knowledge; that errors are frequent in the Inductive processes, and are as much amenable to rules and corrections as errors of Deduction; and that the utility of a Logic strictly confined to Deduction is comparatively small, so much so that writers on the science seldom confine themselves to this department. (For a full consideration of the conflicting opinions as to the Province of Logic, see Appendix, B.)

Thirdly. Logic is a body of Method, or Procedure. It may without impropriety give an account of all known processes that aid the understanding, whether in proving or in evolving truth; provided always that these are of a general kind, adapted to all science or knowledge as such, and not mixed up with the technical specialities of the separate sciences.

There are various admitted uses of Logic that fall under Method. One of these is expressed by Hamilton as 'the rendering explicit in the statement, whatever is implicit in the thought.' In ordinary reasonings, there are frequent omissions or ellipses; and in cases of difficulty or obscurity, these omissions need to be supplied.

The second point belonging to Method is the arranging of an argument or chain of reasoning into the form that best discloses to the mind its conclusiveness or inconclusiveness. This is one great use of the Syllogism. But it is not confined to syllogism. The Inductive canons give a full and precise account of all the possible modes of proving a fact inductively; and by reducing any given proof under its proper heads, we see better what it amounts to. By the same canons, we are also taught what sort of proofs we ought to look out for and produce in any given instance.

Once more. There are certain modes of presenting to the mind all the known facts and premises of a subject, such as to suggest the conclusions involved, and to bring into explicit,

statement, what is implicit and latent. This is a positive aid to discovery.

The Laws of the Association of Ideas may be applied to assist both in Deductive, and in Inductive discoveries. The great end of Deductive Science is, from a given number of data, whether facts or principles, to evolve the greatest number of truths; and the intellectual forces are greatly assisted by adopting certain forms of procedure.

We shall resume, in a final Appendix note, all the bearings of Logical Method, as an Art of Discovery.

DIVISIONS OF LOGIC.

51. In the discovery and verification of knowledge, there are four cardinal operations; one relating to Facts, and the others to the Generalizing of Facts. They are, I. OBSERVATION, including Experiment: II. DEFINITION, or Abstraction; III. INDUCTION; IV. DEDUCTION.

(Observation.

52. If there were rules of observing common to all sciences and subjects, Observation would be a part of the Inductive Logic.

For ascertaining matters of fact, which must be the groundwork of all scientific doctrines, we must have recourse to Observation and Experiment. As regards the material world, this supposes the exercise of the Senses; as regards the subject-mind, it supposes Self-consciousness.

Of all the cardinal processes, Observation is the least adverted to in Logical systems. If it were wholly, as it is in part, a matter of pure intuition, it must be for ever excluded from Logic. In reality, however, it is something more than intuition.

What we term a 'fact,' or an 'observation' is seldom an absolutely single or individual conscious impression. We speak of the fact that high water at Leith follows high water at London by a certain definite interval; but this is far beyond any individual impression upon our senses. It is a generality of considerable compass, the result of the comparison of many separate observations. It is a fact only by reference to some higher generality—to the laws of tidal succession over the globe. There is a process of induction requisite in order to establish such a fact; and all the securities for soundness in

the inductive proofs are called into play. So the fact that the barn-door hen brings forth her young in the egg is an inductive generality; innumerable observations have contributed to its establishment. Only, there are generalities still wider, of which it is an individual constituent; but the difference is merely the difference of lower and higher degrees of generalization.

We come, in the last resort, to observations that are strictly individual. Such are historical incidents; the taking of Jerusalem was an individual fact. So, the details of scientific observation are individual acts of sense and attention. They are not, however, intuitions; for when we say we observe the needle pointing to the north, we include with the impression made on our senses a number of inferences from previous knowledge. It is from previous knowledge that we know we are looking at a needle, and that its direction is north. The simplest observation is thus a mixture of intuition and inference; and our habit of joining the two is one cause of error in the act of observing.

There must be in all observation (of the material world) an exercise of the senses; accuracy of observing is accuracy of sense discrimination. Now the delicacy of the senses is partly natural, partly the result of their exercise upon the special objects. The astronomical observer is trained in the observatory; the physicist and chemist in the laboratory; the anatomist in the dissecting room; the naturalist in the field, or the museum; the medical student in the hospital.

Besides the discrimination by the senses, a good observer is trained to avoid delusive mixtures of inference with observations. He is also indoctrinated in certain artificial rules and precautions for attaining the highest possible accuracy; such as the repetition and comparison of observations, the striking of averages, the elimination of causes of bias in the instruments; to these are added certain mathematical formulæ of Probability, which contribute still farther to the certainty of observed facts. Still, these rules are, for the most part, peculiar to the different subjects.

It is in like manner a special accompaniment of each department to know *what to observe*; to select from a miscellaneous group the circumstances in point. The ongoings of a nation are multitudinous as the sands of the sea shore; the politician or historian knows what to fix attention upon and to record as political facts, the data of political science. The designations applied to the power of political observation are

'appropriate knowledge, a sagacious and discriminating judgment, and analytical reasoning.' No art or rules can impart the intellectual attributes thus described.

Useful illustrations might be given of the errors in observation habitually committed by untutored minds. Still, the best training even for general observation would be a training in some one department. Every educated person should know something of the practical manipulation of at least one of the sciences of observation or experiment—such as a Natural History Science, Physics, Chemistry, or Physiology.

Certain logicians, in dissenting from the inclusion of Induction in the sphere of Logic, have remarked that the rules of Induction must be special to the separate sciences. This is a repetition of the remark just made as to observation. But the cases are not the same. The methods of Induction do not differ in the different sciences, as the methods of Observation differ. Induction in Astronomy is the same as Induction in Chemistry, in Physiology, or in Psychology; the distinctions in the Inductive problem are distinctions that do not divide the Inductive sciences. There may be a common logic of Induction, although not of Observation.

Definition.

53. DEFINITION is a process of *generalization*, confined in its scope to a single property, or a group of properties treated as a unity.

This is the first and simplest of the generalizing processes. When a number of particular things are compared and assimilated on some single property, as round, white, heavy, pungent, the result is a notion, whose expression in any way is Definition. The notion may be complex, or may express several points of agreement, as for example 'life'; but if these are given as united or grouped, they are still regarded as a notion.

The operation of generalizing, with a view to the Notion, assumes a succession of aspects—Classification, Abstraction, General Naming, *Definition*. We assume the last as the representative designation of the whole series.

It is in this department that we see the assimilating and generalizing process in its simplicity and purity. In the department next to be named, generalization occurs, but conjoined with other operations.

Reference will often be made in the sequel to the operation

designated 'Analysis;' and as the process is essentially allied to the generalizing of the Notion, a brief explanation is here given.

Analysis is an adjunct and a result of Abstraction. The separation expressed by the term is of two kinds. The first is the separation of concrete substances, as in the analysis of a water, which separates the saline bodies and impurities contained in the water. This is often a very subtle operation, demanding extreme knowledge, and delicate manipulation. It is, however, an actual separation; the constituents are laid hold of, and exhibited apart.

The second kind of Analysis is the analysis following on Abstraction. It is purely mental: the constituents cannot be exhibited apart. When, by abstraction, we can think of the distinct properties named weight, liquidity, transparency, refracting power, solvent power, we divide, or analyze, in our minds, the concrete called water (pure), into separate properties, although these cannot subsist in separation. Water admits of being classed in many groups; every classification making what is termed an attribute of water. The concrete 'water,' is thus a complexity, an aggregate, or a compound, of many powers; and when these are stated in separation, the concrete is analyzed, abstractively or mentally, not really.

Analysis thus grows out of generalization, being merely a phase or attribute of it. Every act of classifying or generalizing necessarily tends to abstractive separation of this nature. When we class a shilling with round bodies, with white bodies, with bodies of a certain diameter, with bodies made of silver, with bodies stamped as coin—we analyze the concrete shilling into the attributes or abstractions, round, white, size, material constitution, coin.

In the elimination of causes, or productive agents, which is a part of the Inductive problem, a preparatory analysis is essential, in order to isolate in the mind the various antecedents that are to be tested. When a certain impure water is found to produce disease, the water is analyzed in the first instance; and not till the different substances contained in it are found out, can we enter on the enquiry what particular ingredient is the noxious one. This is to apply concrete analysis. Again, when we enquire into the cause of the slaking of quicklime by water, we must analyze in our mind the inseparable properties of water: we must distinguish its solvent property from its chemical affinity, and then proceed to enquire which of these

two, or of any other properties, is the antecedent in the slaking of the lime.

Induction.

54. INDUCTION is the generalization of *conjoined properties*, on the observation of individual instances.

In an induction, we always deal with a proposition, or concurrence of *two* facts or properties: as opposed to the notion, which may consist of a single property. 'Iron takes on' the magnetic property,' is a proposition made up of two conjoined notions—iron and magnetic property. One of these notions singly could be defined, but could not be matter for an Induction.

The circumstance common to Definition and to Induction is generalization. A single isolated instance may be a propositional conjunction, but not an induction. 'This magnet is made of iron' is not an induction: it fails as being only an individual fact.

The largest part of scientific enquiry consists in arriving at these inductive generalizations. The notion is useful chiefly as the constituent of the inductive proposition.

Deduction.

55. DEDUCTION is the application or extension of Induction to *new cases*.

When a general proposition is arrived at, the next operation is to bring it to bear on new instances. By help of the inductive methods, we are satisfied that 'iron is a magnetic substance;' and we apply the proposition, as occasion requires, to individual specimens of iron. Thus the collective iron of the earth comes under the sweep of the proposition; which then indicates the cause, or a cause, of the earth's magnetism.

It is the Deductive process that has been developed into the forms of the SYLLOGISM.

Since Observation is not made a part of Logic, the subject is comprised under the three heads—Definition, Induction, Deduction. There would be no radical inconvenience in expounding the subject in this order, beginning with Definition and ending with Deduction. Probably, if Logic were now studied for the first time, or if the science had followed out its Socratic commencement, this would have been regarded as the natural

order. Circumstances, however, have led to the inverted order—Deduction, Induction, Definition. Although Aristotle himself cultivated all parts of the subject, yet his chief labours were concentrated in the Syllogism, and his followers took up this department to the total exclusion of Induction, and of Definition (as a generalizing process). In the re-introduction of these omitted branches, they have been made to follow, and not to precede the Syllogism.

Another reason for the inverted order is the elementary character of the *formal* Deductive process; it being possible to explain that process without alluding to the Inductive methods for attaining the general propositions.

Under every arrangement, a preliminary portion of Logic is occupied with the elements or constituents of knowledge—the Notion and the Proposition. A full account has to be given of all the diverse forms assumed by these elements in the various departments of information or science.

BOOK I.

NAMES, NOTIONS, AND PROPOSITIONS.

CHAPTER I.

NAMES OR TERMS.

1. There may be knowledge without Language ; but all the truths considered in Logic, are Truths expressed in Words.

The knowledge that guides the lower animals is unconnected with language. They observe by their senses the things about them ; and the observations are remembered in sensible forms. The bush that gives shelter, the herbage for food, the animals to be preyed upon, are known and sought after, by the sole guidance of sense impressions.

Human beings have numerous experiences of the same kind, involving the order of nature, without being connected with words. The child has a large stock of sense knowledge before it can understand and employ language. The skill of the artizan consists, for the largest part, in associations between sensible appearances and movements ; to the stone-polisher, the sight of the surface at once suggests the next blow.

Even in a highly intellectual profession, as the Practice of Physic, the consummation of skill requires a large sense knowledge, passing beyond the scope of language. The physician learns from books, everything that can be expressed in words ; but there are delicate shades of diagnosis that no language can convey, stored up, without verbal expression, in the eye, the ear, and the touch.

Such knowledge, however sufficient for the individual, can be, only to a very limited degree, and with difficulty, com-

municated to others. A *sense* impression, strictly speaking, cannot be directly communicated at all. Indirectly, one individual can be of use to others, by bringing them within reach of the objects that they need to know. The old can carry the young to food, water, or shelter, in the first instance. The instructor in medicine can show the actual cases to the pupil. As regards *movements*, or outward actions, there is the power of imitation, largely possessed by human beings, and to a small extent by animals.

Such communication is obviously restricted to personal intercourse, and, if not so imparted, is lost. The tact and skill of manual arts can be preserved only with the succession of living artizans.

The most signal failure in communication unassisted by names, is in the attempt to convey easily our discoveries of *similarity* or resemblance. In order to teach another man the similarity detected among a number of scattered things, in the point of giving warmth, we should have to direct his attention to the objects one after another, that he might feel the likeness by the actual comparison. How immensely superior is the instrumentality of the names—sun, fire, animal bodies! By the simple process of connecting each of these names, with the common name ‘hot,’ the discovery is made known at once.

This is the primary fact constituting the value of names in general knowledge. A generality is a discovery of likeness, and nothing more. Now, the most rapid and ready mode of imparting all such discoveries is to apply to them a common name. The name ‘tree’ designates a feature of community in a vast number of things; and the use of the name in connexion with all such things makes known the community, the ‘one in the many’ of the Platonic philosophy.

The higher operations of Reasoning often bring together groups of these generalities. A simple product in multiplication—eight times nine makes seventy-two,—contains the following generalities,—eight, nine, multiple, equality, seven, ten, addition, two. Now although these might be severally attainable, by the method of confronting the particulars, yet, without names or signs, the union of them in the multiplying operation would surpass the power of the strongest intellect. By sense alone, we might see that two rows of three, joined in one, would make the row of six; but we should not at a glance discover that seven and eight would make fifteen.

Thus when truths are expressed in language, they can not

only be communicated and discussed; they can also be united into complex propositions, yielding an unlimited fund of derivative truths. It is as so expressed, that knowledge of any kind can be subjected to the tests and methods of Logic.

2. Every portion of knowledge conveyed in language, everything propounded for belief or disbelief, takes the form called, in Grammar, a *Sentence*; in Logic, a PROPOSITION.

A Proposition mentions two things, and is therefore made up of at least *two names*.

We cannot impart, by language, the smallest item of knowledge, without uttering what is called, in Grammar, a *sentence*, which always contains a noun and verb. A sentence is called, in Logic, a PROPOSITION; and is said to consist of a *Subject* and a *Predicate*. The Subject is the thing spoken about; the Predicate the thing said or declared of the subject. The single names 'John,' 'sun,' 'wind,' 'house,' uttered, each by itself, give no information; they constitute neither sentences in Grammar, nor propositions in Logic. They need to be combined, in a certain way, with other names. 'John comes,' 'the sun shines,' 'the wind is lulled,' 'the house faces the sea,'—are pieces of information, sentences, propositions. They all contain at least two words; most of them more than two. In every one of the expressions, we dissect the sense into something spoken about, the Subject—'John,' 'the sun,' 'the wind,' 'the house';—and into something said of each subject—'comes,' 'shines,' 'is lulled,' 'faces the sea.'

We farther remark that any two or more words put together do not amount to an item of information, a sentence, a proposition—something that can be declared true or false, believed, or disbelieved:—'John tree,' 'sun moon light,' 'wind terror tempest,' 'house man street of,'—are not sentences or affirmations. There is a peculiarity in the wording and grammar of all informing sentences. 'Gold yellow,' which as it stands is meaningless, becomes expressive of meaning or information by the help of the word 'is;,' 'gold *is* yellow.' This word 'is' binds the two others into a sentence; grammatically speaking, we call it the verb; logically, it constitutes the *Copula* of the proposition.

While the Sentence in Grammar is divided merely into the two parts,—Subject and Predicate—*subject* 'gold,' *predicate* 'is yellow;,' in Logic, the grammatical predicate is farther divided into the *attribute* of the predicate, 'yellow,' and the

binding word or copula 'is;' the attribute—'yellow' is the logical predicate. A proposition in Logic, then, consists of subject (gold,) predicate (yellow,) and copula (is.)

In affirmations containing but two names, the copula is to be sought in the form of the verb. 'John speaks,' contains a noun and a verb; and the verb 'speaks' has, of its own nature as a verb, the power of affirming. Neither two nouns, 'John lawyer,' nor a noun and an adjective 'gold heavy,' would give any knowledge without a third word as copula; but we have many propositions where a noun and a verb (in a single word) contain a complete affirmation, 'baby walks, food nourishes,' 'Sirius twinkles.'

In these last forms, we can distinguish subject and predicate by our grammatical knowledge; the noun is subject, the verb is the grammatical predicate, and unites in itself the logical predicate and the logical copula of affirmation. Also in such forms as 'gold is heavy,' we are guided by grammar. We know that an adjective, as 'heavy,' is never a subject, and must therefore be the predicate. The noun can be both a subject and a logical predicate;—'gold is a metal,' 'Cæsar is emperor' contain each two nouns, one being subject and the other predicate; which is which may be usually determined in English by the order; the subject being given first. When the order is inverted for Rhetorical effect, we must judge by the meaning and the context.

The fact cannot be too soon laid to heart, that the predicate is usually *larger in meaning than the subject*; it applies to many other things besides the one spoken of at the time. 'Gold is heavy,' but not the only heavy thing; 'heavy' applies to other substances besides gold. 'Woody fibre is not fit to eat,' leaves us free to affirm that there are many things not fit to eat, as well as woody fibre. Hence, subject and predicate in affirmation, are *not necessarily co-extensive*; in point of fact, they are *very seldom* co-extensive.

3 There are various motives or reasons for commencing Logic with an examination of Names.

(1). It has now been seen that a Proposition, the final constituent of Logic, the logical form of all knowledge, is made up of Names. The characters of propositions, therefore, cannot be given without referring to their component names.

(2). In the use of Names are involved numerous sources of error,—pitfalls and snares; and it is one function of Logic to give warning of these.

(3). An examination of the existing vocabularies of mankind is the readiest clue to the universe of existing things. A language, if fully developed, indicates all the things that the persons speaking it have taken notice of; these may or may not be everything that the world contains, but they are everything brought to light by the combined observation of many men through many ages. Now, it is found useful, in laying down the scheme of a comprehensive Logic,—a code of Evidence and of Methods for all kinds of knowledge—to survey and reduce to heads the whole universe of ascertained things. The vocabulary of the most advanced and cultivated people, or of several peoples combined, is the best available aid to this operation.

In an advanced language, we find names for the heavenly bodies, and their revolutions, and changes; names for large objects on the earth—sea, mountain, river, &c.; names for separate material substances—water, stone, iron, gold, wood, ivory; names for powers and forces,—wind, weight, heat; names for living bodies—plants and animals; names for the bodily parts and functions of human beings; names for mental functions—pleasure, pain, will, thought; names for the social facts of humanity—king, law, punishment, property, crime; names for the numerous exercises and functions of mankind—husbandry, trade; and so on. Now the names give the clue to the various objects named. Again, we have names and forms of speech indicating *agreement* among things—generic or common-words, as star, solid, heat, power, pleasure—which show us that natural facts frequently *recur*. Farther we have names that imply other names;—ruler-subject; up-down; whence we learn that the world contains mutually connected things.

4. A name is defined, in the first instance, ‘a mark attached to a thing to enable it to be spoken about.’

In giving names to objects, the end primarily sought is communication and discourse. Once invented, names have the additional function of aiding the solitary thinker, in recalling, fixing, and arranging his thoughts.

It is remarked by Mr. Mill, as a corrective to the unguarded views of Locke and others, that names are the names of Things, and not of the Ideas of things. The word ‘sun’ is the mark of the object called by that word, and not simply the name of our thought or idea. To suppose that names are names of ideas alone is a species of idealism, confounding together the object and the subject. The Thing itself (if an object) is determined by our sensa-

tions, or what we call our experience of actuality; the Idea is purely subjective; it is a mental element strictly so called.

5. For the purposes of Logic, Names have regard to GENERALITY and to RELATIVITY; in correspondence with the two foundations of knowledge—*Agreement* and *Difference*.

Names may be variously classified. They may be divided *philologically* into languages, as English, French, Hebrew. They may be divided for *rhetorical* purposes into plain and figurative; the figurative class containing species—Hyperbole, Irony, &c.,—opposed to Logic, as departing from truth for the sake of the feelings.

There is also a division of Names under *grammar*, namely, the Parts of Speech, which may be looked upon as in great part a logical division. Thus, the Noun may be always the subject of a proposition, and is often a predicate. The Adjective has two logical functions;—it may be, and frequently is, a predicate; and, secondly, it is the *specifying* designation of a genus expressed by a noun; man (Noun), *genus*, white (Adjective) man, *species*. The Verb has the important logical function of affirmation or predication; there can be no proposition without a verb; 'fire *burns*,' 'honey *is* sweet.' The remaining parts of speech possess no logical function.

NAMES CLASSED ACCORDING TO GENERALITY.

6. In classing Names, with reference to GENERALITY (or Agreement), the fundamental distinction is between *Singular* Names and *General* Names.*

The process of generalization, through the tracing of agreement, is a thoroughly scientific or logical process. Now, whether for a general *notion* (as 'liquid'), or for a general *proposition* ('liquids find their level'), the names employed are

* In the foundations of knowledge, Discrimination or Relativity may be supposed to have the priority; we discriminate first, and trace agreements in difference afterwards. On this view, the classification by Generality might properly precede the classification by Relativity. In reality, however, we cannot treat either without the other being implicated; the relative couple, light-dark, is understood by us only as generalized upon many recurrences of the transition: we do not go back, for our typical notion of the phenomenon, to the first occasion when we experienced the shock of transition, or before we had identified several recurring shocks. There is, therefore, no special disadvantage in beginning with generality: we being aware that there could be no notion of either individual or general, without prior shocks of discrimination or relativity. Whichever of the two facts is under consideration, the other must be tacitly supposed.

general names. Moreover, the individuals that have to be identified and compared in order to the generals, must also have their names as individuals,—‘the Rhine,’ ‘the Caspian sea.’

7. A Singular or Individual Name is a name applicable to one thing. A General Name is applicable to a number of things, in virtue of their being similar, or having something in common.

Xerxes, Bucephalus, Sirius, Teneriffe, the Alps, England, Rome, Notre-Dame, Koh-i-noor, are Singular names; they designate each one individual object.

Man, horse, star, mountain, kingdom, city, building, gem, are general names; they apply each to an indefinite number of things having a certain likeness or community among themselves.

The Singular Name may be of various forms. One form, exhibited in the above examples, is a single meaningless mark or designation appropriated to the thing. ‘Xerxes,’ ‘Sirius’ have no function but what might be served by any other distinctive utterance applied to the objects indicated. A modification of this form is seen in the many-worded designations of individual men and women, John Davidson Ross; Maria Anne Louisa Brown; David Smith, of George Street, York. A plurality of words must be resorted to, because John, Maria, Brown, &c., are used in naming a great many individuals, and are therefore not distinctive. Such names furnish the least possible information about the persons named. They do not necessarily indicate human beings; horses, dogs, ships, &c., receive designations from the same class of words.

Another form of the singular name is seen in such examples as ‘the reigning Pope,’ ‘Her Britannic Majesty’s minister at Berlin,’ ‘the discoverer of America,’ ‘the high-priest of Baal,’ ‘the youngest of the family,’ ‘the pinnacle of Europe,’ ‘the vault of heaven.’ These are severally applicable to individuals, but they suppose previous generalities, combined so as to restrict the meaning to definite individuals. They are significant although also singular; and the significance grows out of the generalities.

Collective names, as nation, army, multitude, assembly, universe, are singular; they are plurality combined into unity. But, inasmuch as there are many nations, armies, assemblies, the names are also general. There being but one ‘universe,’ that term is collective and singular.

Names of Material—earth, stone, salt, mercury, water,

name,—are singular. They each denote the entire collection of one species of material. If Space and Time be not regarded as abstractions, they fall under the present class.

8. General Names are said to be *Connotative*; that is, they denote objects, and *connote* or imply attributes, or points of community among objects.

As a mere mark, a name has no power beyond simply denoting, or pointing out its object; Sirius suggests the star of that name; London has no other function than to make us think of the object named. But the general name, the result of assimilation, *denotes* the individuals, and *connotes* or implies a certain similarity among them, in other words, a common attribute. The word 'star' denotes any star in the firmament, and implies or connotes the similarity pervading the stars; the word 'metropolis' is the name denoting London, Paris, Berlin, and also declaring that all those separate objects have points of resemblance; the resemblance is the common attribute of the things, and the *connotation* of the general name.

All Class names, therefore, being general names, are connotative names:—man, animal, plant, tree, metal, mountain, sea, kingdom, government, factory, circle, virtue.

Besides, the general or class nouns, Adjectives are to be held as connotative:—for example, white, square, wise, virtuous. These are generalized names; they are given to a plurality of things agreeing in a certain way. They each denote particular objects (the noun being supplied); they *connote* or imply a community in these objects. They are significant and not meaningless names.

Adjectives are obviously products of the generalizing process no less than class nouns. The same generalization is often expressed both as a noun and as an adjective,—circle, round or circular; colour, coloured; weight, weighty.

The limitation to this practice belongs to the nature of the things. The function of an adjective is to narrow the application, and increase the meaning of a noun; 'wise men' are fewer in number, and more numerous in attributes than men. Now, in order that a noun may take on the whole meaning of an adjective, that meaning must be a limited one; it must be expressive of only one or a few attributes. 'Men' can take the qualifications signified by the adjectives 'wise,' 'old,' 'tall,' 'virtuous.' If, however, we were to coin an adjective from the class 'horse,' there are no objects in nature that could take, in addition to their own attributes, all those possessed by horses. When adjectives are formed from such classes—commonly called natural kinds—they are used only in a select or partial meaning. 'Golden' means either made of gold,

or possessing the salient and striking attribute of gold; 'feline' signifies only one single feature of the genus 'fel'; 'human' is some peculiar attribute of man.

Sometimes a general name is explained as being the name of a *class*; 'man' the name of the class men. But the word 'class' has two meanings—the class definite, and the class indefinite. The class *definite* is an enumeration of actual individuals, as the Peers of the Realm, the Oceans of the globe, the known Planets. The individuals of these classes have a certain likeness or common character; while, in addition to this, they are all known and enumerated. The question whether a certain object belongs to the class, might be settled in two ways; first, by its possessing the class likeness, secondly, by its being found in the enumeration. The shortest way of ascertaining whether a given person is a peer of the realm would be to look for his name in the Peerage. At all events, this dispenses with the method of judging by means of class marks.

The class *indefinite* is unenumerated:—such classes are stars, planets, gold-bearing rocks, men, poets, virtuous. These classes contain individuals known and many more unknown. There is no complete list whereby to test any supposed individual. The sole criterion is the class attribute or likeness. Whether a newly-discovered heavenly body be a star or a planet is to be decided by finding its characters. If it is a fixed body, we class it with stars, if it circles round a fixed star, we class it with planets.

In this last acceptance of the word, class name and general name are identical. The class name denotes an indefinite number of individuals, and connotes the points of community or likeness. The general name does the very same thing. The designation 'wise men' is a class name and also a general name. But in the acceptance of an enumerated and finished list, the class name is not the same as the general name; it provides an additional, and exceptional test of the claims of individuals to belong to the class. 'Thales is one of the seven wise men' exemplifies the class definite; 'Socrates is wise' sets forth the class indefinite, known only by the meaning of the general name.

9. The contrast designated by the words 'denote' and 'connote,' corresponds to Hamilton's distinction between quantity in *Extension* and quantity in *Comprehension*.

The denotation of a general term, the individuals that it

applies to, is designated by Hamilton, its *Extension*, or extent. The denotation or Extension of the term 'man' is the whole population of human beings. The connotation or Comprehension is the community of attributes, or points of agreement, making up the characters, marks, or definition of men—animal life, anatomical peculiarities, mental endowments, &c.

The two facts—denotation or extension, and connotation or comprehension—are reciprocally opposed; the greater the one the less the other. The term 'animal' has a greater denotation or extension than the term 'man'; it includes all men, and the population of brutes besides. It has so much the less connotation, or comprehension; it connotes only the points common to animals, which are much fewer than the points common to men;—animal life in general, without distinctive organized forms. On the other hand, the term 'wise men' denotes less, has less extent, than the term men; it applies only to a selection of men. It connotes or comprehends all the more; to the connotation of men it adds the attribute connoted by 'wise.'

Mr. De Morgan has dwelt at great length, and expressed in a variety of forms, the distinction between Extension and Comprehension—Breadth and Depth,—and has followed it out, like Hamilton, into syllogistic forms.

He remarks that Terms are used in four different senses. Two of these, he calls *objective*, as directed to the *external* object. The first are terms expressing an individual standing alone, or out of all connexion or relationship with any other individual; as John, man. The second, the name of an *individual quality*, forming part of, or residing in, the individual object, as the term 'human,' or as 'animal,' when applied to man. The author considers that the ordinary syllogism has reference to these terms, which he calls terms 'of the first intention,' and also *arithmetical*. The usual form of a proposition is to declare some objects to be included in, or to be excluded from, some other objects; or to affirm or deny of them some quality in the form now stated—'men are animals,' 'kings are human.'

The two other senses of Terms are called by the author *subjective*. The first is to represent a *class*, or collection of individuals, named after a quality common to all: these are Mill's connotative class names. The second represents the *attribute* of the class apart, in other words, the abstraction as conveyed by the abstract name. In short, in these subjective meanings, explicit notice is taken of the fact of 'generality' or 'generalization;' the one in the concrete and the other in the abstract designation.

It may be remarked on the distinction between these objective and subjective meanings, that it hardly involves any serious difference. Unless the objective terms were confined to proper,

names, they are terms having generality, and that generality (perhaps more expressly brought into the foreground) is all that is indicated by the subjective terms for class and attribute. Take the author's illustration of all the four—man, human, mankind, humanity—the two first objective, the two second subjective; the difference between 'man' and 'mankind' is impalpable; while 'humanity' is merely the abstract noun of the adjective 'human.'

The real distinction is between the class and the class attribute.

For 'extension and comprehension,' Mr. De Morgan employs the terms 'extent' and 'intent,' also 'scope' and 'force.' He farther draws attention to an important distinction in the modes of combining terms of extension and terms of comprehension respectively. When terms of extension are combined, as 'man' and 'brute,' there is an arithmetical summation of individuals; this he calls *aggregation*. When two terms expressing attributes combine, as 'white' and 'polished,' it is not an arithmetical sum or aggregate, but a joint inherence of quality in a common subject; to this he applies the name *composition*. He remarks that we have not a good English designation for the separate parts of a compound, in this last sense. The word 'part' refers to extension. The words 'constituent' and 'element' are a nearer approach to the idea, but do not exactly hit it.

Boole, in his system, expresses aggregation by the sign of addition, man + brute, $x + y$; and composition by a product, white \times polished, $x y$; and conducts his manipulation throughout in conformity with these suppositions.

10. The final result of the generalizing process is the **ABSTRACT NAME**. This is an elliptical form of speech, highly useful, but also greatly abused.

Such names as motion, weight, breadth, roundness, whiteness, melody, sweetness, roughness, polarity, wisdom, justice, beauty, are called *abstract* names, as signifying qualities or attributes without reference to the things that possess the qualities. They seem to separate the points of community of agreeing objects, from the objects themselves, an operation impossible in fact, and even in thought, but supposed, by a kind of fiction, to be possible. They give the meaning expressed by the connotation of the corresponding class designations—moving things, heavy things, broad, round, white, &c., but they drop entirely the denotation.

The abstract name, although occurring in all languages, is not absolutely required for ordinary speech; nor indeed for science. The meaning to be conveyed can always be given, although not so shortly, by means of general or class names. The name 'motion' expresses what is meant by 'moving things;' the farther effect of it is to limit the consideration

to this one feature of the things in question; it amounts to saying 'moving things in so far as moving,' or with reference to the one circumstance common to them all, and not to any other circumstance that may attach to particular individuals. So 'justice' expresses the same meaning as 'just actions;' the only existing fact corresponding to the term is the class 'just actions.' There is no such thing in the universe as justice by itself; we cannot point to a disembodied justice. The term signifies 'just actions,' with a peculiar stress or emphasis put upon the features of agreement; 'just actions in so far as just, or viewed solely with reference to their being just.' The proposition '*Justice commands respect*,' is the same proposition as 'just persons are respected persons,' with a more emphatic indication than the class names seem to give, that the causation refers solely to the points common to 'just persons,' and to 'respected persons.' 'Just persons so far as just are respected persons so far as respected.' 'Beauty gives pleasure' is equal to 'beautiful things (in so far as beautiful) are things pleasant (in so far as pleasant).' There is no 'beauty' in the abstract giving 'pleasure' in the abstract; such a supposition is the old error of Realism, scarcely yet extinct. 'Mind is the cause of force' can mean only 'beings possessing mind, in so far as possessing mind, are the cause of moving things considered as moving.' 'Mind' is inseparable from certain actual beings called persons, beings mentally endowed, &c.; and 'force' is an abbreviation for moving things, the cause of other moving things, in so far as moving.

A great power of abbreviation is given by abstract terms, which is probably the motive for introducing them so largely into common speech. This is apparent from the circumlocutions necessary for avoiding them.

The abuse of abstract names is exemplified in the almost irresistible tendency they have to suggest the existence of things in the abstract. We are led to suppose from the use of the terms Time, Space, Mind, that there is something in nature called Time, apart from things enduring; something in Space different from things extended and the free movements of extended things; something named Mind, distinct from beings exerting mental functions.

An important logical exercise, for detecting the fallacies nursed under abstract names, is to translate abstract propositions into the equivalent propositions made up of general names, not abstract.*

* If the student of philosophy would always, or at least in cases of importance, adopt the rule of throwing the abstract language in which it

In contrast to abstract names, all general names, or class names, are termed **CONCRETE** names: they express the agreement among things, not as an impossible detached fact, but in the actual state of the case, namely, as the things that possess the agreement. All class nouns, as man, tree, star, and all adjectives, as brave, tall, lustrous,—are *concrete* general names. Every connotative name is thus a concrete name.

We must not confound, as is sometimes done, a general name with an abstract name. A general name is opposed to an individual or singular name; an abstract name is opposed to a concrete name, whether general or individual. The abstract 'whiteness' is opposed to the general designation 'white things,' and through it to every particular white thing.

The Abstract name cannot possess the double function of the general name,—denoting a thing and connoting the similarity of things; it may be said, as by Mr. Mill, to denote the similarity, or the common attribute, and to connote nothing. There is, however, nothing gained, anywhere in Logic, by such a designation. The Abstract name is the last product of generalization; alike the facility and the snare of general expression.

It is a consequence of the generalizing process that there should be names of lower and higher generality, as Englishman, European, man, animal, organized being; circle, curve, geometrical figure, extended thing. These successive generalities play a great part in science, and lead to many technical designations which have to be considered in Logic; but their suitable place is in the following chapter, on the Notion, or Concept.

11. The second group of Names, viewed for Logical ends, embraces those connected with **RELATIVITY**.

The essential Relativity of all knowledge, thought, or consciousness, cannot but show itself in language. If everything that we can know is viewed as a transition from something else, every experience must have two sides; and either every name must have a double meaning, or else for every meaning there must be two names. We cannot have the conception 'light,' except as passing out of the 'dark;' we are made

is so frequently couched into a concrete form, he would find it a powerful aid in dealing with the obscurities and perplexities of metaphysical speculation. He would then see clearly the character of the immense mass of nothings which constitute what passes for philosophy.' (Bailey's Letters on the Mind, vol. ii. p. 159.)

conscious in a particular way by passing from light to dark, and from dark to light. The name 'light' has no meaning without what is implied in the name 'dark.' We distinguish the two opposite transitions, light to dark, and dark to light, and this distinction is the only difference of meaning in the two terms; 'light' is emergence from dark; 'dark' is emergence from light. Now, the doubleness of transition is likely to occasion double names being given all through the universe of things; languages should be made up, not of individual names, but of couples of names. When we refer to the actual case, we find a very great prevalence of couples, but we can hardly call it universal. We have such instances as heat-cold, motion-rest, up-down, light-heavy, thick-thin, hard-soft, rich-poor, life-death, parent-child, ruler-subject; and we must enquire how far the system extends, and, if short of universality, why it is so.

12. The great distinction of Names founded on Relativity is expressed by POSITIVE and NEGATIVE names.

No one designation exactly suits the principle of universal relativity. The couple 'Positive and Negative' is the best we have, but the term 'negative' inclines too much to the idea of deficiency, or absence of a quality, without the presence of a corresponding opposite. Now the negative of a real quality is as much real as the positive; North and South, have an equally good title to positive existence. Heat and cold, or the transitions cold-heat, and heat-cold, are equally real or present experiences.

The terms 'Relative' and 'Correlative' are also too limited for the purpose; they are too much confined to complex relationships, as, parent-child, teacher-scholar, mover-moved.

Of these two couples, the one most easily adapted to the universality of relation is the first—'Positive and Negative;' which we shall adopt with the understanding that 'negative' has always a real existence, no less than 'positive.' So explained, it may be stretched to the whole length of universal relativity. Under 'Relative' and 'Correlative,' will be explained certain special relationships, growing out of the complicated arrangements of the world.

Mr. Mill expresses the nature of Positive and Negative in the following terms:—'To every positive concrete name, a corresponding negative one might be framed. After giving a name to any one thing, we might create a second name which should be a name of all things whatever, except that particular thing or things. These negative names are employed whenever we have occasion to speak collectively of all things other than some thing or class of things. Thus *not-white* de-

notes all things whatever except white things ; and connotes the attribute of not possessing whiteness.' 'Names which are positive in form are often negative in reality, and others are really positive though their form is negative. The word *inconvenient* for example, does not express the mere absence of convenience ; it expresses a positive attribute, that of being the cause of discomfort or annoyance. So the word *unpleasant*, notwithstanding its negative form, does not connote the mere absence of pleasantness, but a less degree of what is signified by the word *painful*, which is positive. *Idle* on the other hand, is a word which, though positive in form, expresses nothing but what would be signified either by the phrase *not working* ; or by the phrase *not disposed to work* ; and *sober*, either by *not drunk*, or *not drunken*.'

Thus far Mr. Mill. Mr. de Morgan carries the distinction to the length of a mode of universal relativity. He says :—' Let us take a pair of contrary names, as man and not-man. It is plain that between them they represent everything imaginable, or real, in the universe. But the contraries of common language usually embrace, not the whole universe, but some one general idea. Thus, of men, Briton and alien are contraries : every man must be one of the two, no man can be both. Not-Briton and alien are identical names, and so are not-alien and Briton. The same may be said of integer and fraction among numbers, peer and commoner among subjects of the realm, male and female among animals, and so on. In order to express this, let us say that the whole idea under consideration is the *universe* (meaning merely the whole of which we are considering the parts) and let names that have nothing in common, but which between them contain the whole idea under consideration, be called contraries *in, or with respect to, that universe*. Thus the universe being mankind, Briton and alien are contraries, as are soldier and civilian, male and female, &c. ; the universe being animal, man and brute are contraries, &c.'

Mr. de Morgan here supplies what is requisite to the precise definition of Positive and Negative. It is not strictly correct to say that 'not-white' means everything in nature except white things ; a more limited universe is supposed at the time, probably the universe 'colour ;' and the meaning of not-white is black, red, green, yellow, blue, &c. Sometimes a still smaller universe may be intended, the universe of white, black, and the shades of grey ; the prismatic colours being excluded ; in which case not-white means black and grey.

When a term is ambiguous, one mode of rendering it pre-

cise, is to name the opposite of what is meant. The term 'civil' has many meanings; it is opposed to natural, to military, to ecclesiastical, to uncivil or discourteous, and so on. The same purpose is served by stating what higher universe is present to the mind of the speaker. If the universe be the condition of human beings in relation to one another, 'civil' means organized into human society; if the universe be the departments of government, 'civil' is known to exclude military and ecclesiastical; if the universe be manners or address, civil is understood in that connexion.

Thus of the three things—the universe or genus of the speaker, the positive, and the negative—we cannot know one without knowing the others. Any ambiguity in one is remedied by stating a second; it matters not whether that second be the contrary or the entire universe. In common speech, we are usually able to assign the universe from the context or occasion. In discussing the origin of human society, we see that the words 'civil' and 'natural' are employed to divide the universe of man's condition in respect of society. When we do not know the subject of discourse, we are still made aware of what a term means, if the opposite happens to be given, as 'civil,' 'not rude.'

13. In those cases, where a universe contains but two members, the one is the complete negative of the other. This is the most marked form of contrariety.

Heat-cold, light-dark, high-low, straight-bent, good-evil, pleasure-pain, virtue-vice, health-disease, man-brute, are complete and emphatic contraries; the negative of one member is the affirmation of the other; the affirmation of one, the negative of the other.

14. When a universe, or higher genus, contains many members, the contrariety, although no less real, becomes diffused.

'Red' in the universe *colour* is not negated by any single colour, but by a plurality of colours. If we are dividing colours according to the Newtonian spectrum, 'not-red' means six colours. In a full enumeration of shades of colour, 'not-red' would be a list of many scores of individuals. The contrariety is then diffused and pointless. 'Not an Englishman' leaves us in a wide sea of possibilities; the universe being natives of different countries.

15. Language contains various modes of expressing opposition or negation.

(1) In certain prominent instances, separate names are given to the contraries; as in many of the examples already quoted. Our language contains perhaps some hundreds of couples of contrary names: young-old, wise-foolish, brave-cowardly, rising-falling, good-evil, sweet-bitter, rough-smooth, health-disease.

(2) There are certain general modes of stating negation. The chief is the prefix *not*:—*not-cold*, *not-well*, *not a fish*, *not-metal*, *non-electric*.

The prefixes '*un*,' '*in*,' and the suffix '*less*,' are also used: *unknown*, *incomprehensible*; *heedless*, *blameless*.

The purpose is also served by various circumlocutions — '*everything but*,' '*all but*,' '*all that remains when one is withdrawn*.' These last forms express accurately the real process of negation when disguised by plurality of contraries; a universe is assumed, the given positive is subtracted from that universe, and what remains is the negative or opposite. '*All the simple bodies except the metals*' explains the meaning of *not-metal*, in the universe '*simple body*.' '*All the parts of speech except the noun*,' is the full rendering of '*not a noun*,' '*not-noun*.'

16. The Negative of a real property or thing is also real.

If a negation be simply the remainder when one thing is subtracted from a universe containing more than one, such negation is no less a positive reality than the so-called positive. In fact, positive and negative must always be ready to change places; positive *up*, negative *down*; positive *down*, negative *up*.

There are certain circumstances, where one side seems to be positive, by a special propriety; as when we express fulness, abundance, or presence, as opposed to deficiency, or absence. '*Wealth-poverty*,' '*debt-credit*,' '*plus-minus*,' '*full-empty*,' '*strong-weak*,' '*living-dead*,' '*knowledge-ignorance*,' '*fruitful-barren*,' '*something-nothing*,'—these seem to give us on the one side a truly positive conception, on the other side, a truly negative; the reversal of the terms would seem harsh, unnatural, distorted. Yet, in all such cases, the negation is a real and definable phenomenon; a genuine experience of the human mind, although, in most instances, a less agreeable experience. The position of being in debt is a real fact or

state, with characteristic features; there is an assignable universe, the universe of pecuniary circumstances; we subtract from that total the cases called being 'out of debt,' 'solvent,' and we find as a remainder cases of 'being in debt;' the two are mutually opposed; we might call either positive, and the other negative. Any awkwardness in the free transposition of the epithets arises from the imperfection already noticed as attaching to those epithets, considered as names for universal relativity. They are frequently used with more limited and special associations, such as to give a greater seeming propriety to the employment of 'positive' for the conditions expressed by abundance, wealth, credit, strong, pleasurable, good, than to the employment of 'negative' for those conditions.

The highest universe of all must contain at least two things, mutually explaining, and equally real. This remark is necessary, because a fallacy is often committed by using the forms of language where there is no longer a reality to correspond. Thus matter-mind, or more correctly extended-unextended, —object-subject—signify a real couple, mutually explaining. The denial of matter, extension, or the object-world, is the affirmation of mind, the subject-world. Up to this point, we are in the region of real and actual experience. There is a transition familiar to us, between certain states of consciousness called matter, and other states called mind; we know both, by mutual contrast; while our *knowledge* can ascend no higher. Still, *language* can take a flight beyond. We can in *words*, sum these two facts together—mind and matter, subject and object; we can even use a single term as the equivalent of this sum—Universe, Existence, Absolute; but our knowledge is not advanced by the step. There is nothing correlative to the supposed universe, existence, the absolute; nothing affirmed, when the supposed entity is denied. Matter we can conceive, because of its real opposite, mind; but 'existence' has no real opposite.

Granting for a moment, that there were such a thing as non-existence, to give reality to existence, what is to prevent us from summing these two together, giving a name to the sum, and insisting on the reality of this new entity, with a correlative reality; and so on without end? We must obviously stop somewhere; and the proper point is the highest *couple* that generalization can carry us to. This is to conform to the essential relativity or doubleness of knowledge. An absolute unity is not knowledge, but an unmeaning phrase.

17. Many Special Relationships, apart from universal relativity, are involved in the processes of nature, and in the relationships of living beings. From these, we have numerous relative terms.

In the act of communicating motion, there is a thing moving and a thing moved, something striking, and something struck. In support, there is a supporter and a thing supported. Attraction and repulsion require two things; the attracting and the attracted. Heat and light emanate from some body and operate upon other bodies. Acid is relative to alkali or base; both to a neutral salt.

Procreation implicates parents and offspring. Male is correlative with female; the name 'male' has no meaning by itself; we must understand 'male' and 'female' by the same indivisible act of intelligence. The fact that they express is a complex fact; both parties are concerned in it; the part of one cannot be separated from the part of the other.

'Lock' and 'key' are correlative terms of this kind. We cannot understand or explain a key without the mention of a lock, nor a lock without a key.

The complex structure of human society contains many situations, where two parties mutually enter. Such are sovereign-subject, master-servant, buyer-seller, debtor-creditor, accuser-accused, teacher-pupil, doctor-patient, churchman-dissenter. These are cases, not of universal, but of special, relativity, and deserve to be considered apart from the more fundamental relationships inherent in knowledge.

All active verbs are correlative from the very necessity of their structure. An agent supposes something to act upon; unless viewed in act, it has no meaning. A conqueror that never conquered anybody is an absurdity.

It is commonly said, with reference to the great problem of the Perception of a Material World, that knowledge 'supposes a mind knowing, and a thing known': which is interpreted as proving that there is a mind apart from matter. In truth, however, it proves only, that, in the act of knowledge, as in every other act, there is a mutual participation of two things. Whether these things can exist as separate, detached, and independent entities, is a distinct enquiry.

18. The meaning of every object of knowledge enlarges with the enlargement of its negatives or contraries.

Gold, in the universe 'simple body' means the opposite,

or exclusion of the other sixty-two simple bodies. If ten more elements be discovered, there will be ten more exclusions or opposites. 'Health' to a rustic means the absence of a certain number of familiar diseases—catarrh, rheumatism, dyspepsia, measles, &c.; to a hospital nurse, it has a still wider meaning; to an institutional writer on Medicine, it means the exclusion of upwards of a thousand diseases.

There is no escape from the principle of universal relativity. There is no possibility of mentioning a thing, so as to be intelligible, without implicating some other thing or things, equally intelligible. One might suppose that a chair is an absolute and unconnected fact, not involving any opposite, contrary, or correlative fact. The case is quite otherwise. The chair is immediately opposed to vacuity, and to the physical and mental condition of the person suffering from its absence. It may, according to the circumstances, have a still greater compass of opposition, and so a still wider meaning; it may be opposed to a table, a bed, a footstool. It may have still farther oppositions; the reference may be to the universe 'seat'; in which it would be opposed to a 'stool,' 'a bench,' a 'sofa,' 'an ottoman,' &c. The full meaning would then be I do not want a 'stool,' 'sofa,' &c., but a *chair*.

CHAPTER II.

CLASSES, NOTIONS, OR CONCEPTS.

1. These designations signify generalization applied to *single* properties, or to groups of properties regarded as units or wholes.

The contrast is to Propositions, which are generalized *couples*, with the affirmation (or denial) of coincidence.

We may identify and generalize a number of things under a *single* point of community, as 'round,' 'heat,' 'polarity.' In the concrete, these generalities are named classes—'round things,' 'hot things,' 'polar things.' When the point of community is spoken of in the abstract,—'roundness,' 'heat,'

polarity,—the abstraction is called a general notion, a general concept, and often simply a notion, or concept; the terms 'notion' and 'concept' being regarded as more applicable to a generalized property, than to a single concrete object. The phrase 'abstract idea' is an equivalent expression, for the common property of a class.

* It is impossible to confound these classes, or notions, having only a single feature in common, with propositions, which must have at least *two* things. But many classes have more than one feature in common; as 'metals,' which agree in four or five points. The class 'man' has a still greater number of points of agreement. In such instances, the distinction between the class, or the general notion, and the proposition, appears to be done away with. It no longer turns upon the number of common properties, but upon the manner of expressing their conjunctions. In the class, the conjunction of the properties in a group is assumed; there is no question raised, as to whether they are conjoined. In the proposition, this is treated as open to doubt, and the doubt is met by a positive assurance, in the form of a distinct affirmation, backed up, if need be, by proof or evidence.

The following are examples of the generalized Proposition, involving two notions linked together by affirmation (or disjoined by denial). 'The circle contains the largest area within a given circumference'; 'heat is convertible into mechanical force'; 'the metals are the bases of salts.' In every one of these there are two distinct general classes or notions; the class 'circle', with the class or notion 'largest area in a given circumference'; the class or notion 'heat' and the notion 'convertible into mechanical force'; the class 'metals', and the class 'bases of salts.' But the existence of two notions does not exhaust the force of the proposition. There is farther the information that the two in each case do, or do not, go together. A hearer is supposed to be in ignorance or in doubt as to whether the notions 'circle' and 'maximum of area' are coincident; and the proposition sets this doubt at rest, so far as affirmation can go.

Obviously, it is only the *affirmative* or conjunctive proposition that can ever be confounded with the double-propriety class; the negative proposition declares the *disjunction* of things.

The nature of the Class, Notion, or Concept, has been unavoidably brought out under 'Names,' and more especially under names grounded on generality.

2. Many classes are based on a single point of community; otherwise expressed by saying that they possess only one attribute; as white, hard, long, extended, round, polar, hot, pleasure, multitude.

'White,' being a single, indivisible impression on the mind, the things that agree in it, and in nothing besides, are classes based on one point of community; they have only a single class attribute. Such classes are numerous. The properties—transparent, hard, soft, elastic, brittle, long, square, hot, liquid, air, simple body, pleasing, just, powerful—are single features of agreement; there are communities of things comprising these several individual features, and no others; and they are all treated as simple effects. *

3 There are classes formed upon more than one, but yet not many, points of community.

A good number of classes have two points in common. A house is (1) an artificial erection, (2) for the purpose of sheltering living beings or things belonging to them. A town is (1) an assemblage of inhabited buildings, (2) under a common government. A magnet is a body (1) attracting iron, and (2) polarized. .

As an example of a triple-propriety class, we may cite 'Mind,' which comprises three distinguishable functions—Feeling, Will, Intellect. Chemical Affinity has also a triple definition;—definite proportions, change of properties, production of heat.

The long received definition of 'Inflammation' enumerates four properties—Heat, Redness, Swelling, Pain.

4. There are certain Classes grounded upon a large and indefinite number of common features. These are termed, by pre-eminence, real Kinds, *Infimæ Species*, lowest Kinds.

* The singleness, in some of these instances, is relative to the usual mode of defining by reference to a higher genus with a statement of the specific difference (*per genus et differentiam*). Thus 'round' is a plane figure with a special mark (given in the definition of the circle). The inclusion of the generic attributes of the plane figure (Extension and Figure) along with the special difference would make roundness, or the circle, a plural notion. 'Pleasure' is of the genus 'feeling,' with a specific difference, which is a single property; the genus and difference combined would give two properties. 'Extended' is absolutely single, being the highest genus of all, on the object side. . For the complete theory of Definition, this explanation is material; in the present connexion, notions may be held as single, whenever the specific difference, usually assigned in defining them, is single. In many notions, this specific difference is complex.

The simple bodies of Chemistry—Oxygen, Sulphur, Silicon, Sodium, Tin, Gold, &c.—have each a series of distinctive properties. The number actually known is considerable; and there may be many unknown. There are from ten to twenty properties given in the usual account of Oxygen; and about as many in the description of Iron, and of Gold.

‘Again, in the Vegetable world, we have classes founded on a great number of common properties. The classes termed ‘Species,’ in the peculiar sense of Species in the Natural History Sciences, have a great many characters;—many common peculiarities in form, in mode of growth and development, chemical products, &c. A full account of the British Oak would extend to at least twenty or thirty characters.

Still more in the Animal Kingdom, have we the aggregation of many features in the same class. The properties common to the species ‘Elephant’ are very numerous; a full enumeration of the bodily and mental peculiarities of the species would require perhaps fifty to a hundred designations. The common properties of the class ‘man’ are still more numerous.

It is in these three great departments—the Mineral, Vegetable, and Animal Kingdoms,—that we have the culminating instances of plural properties. The greatest complications known apart from these do not pass beyond a small number of properties. The most intricate disease, for example, can usually be characterized by not more than five or six distinctive features.

5. Classes are of *higher* or *lower* GENERALITY: whence arises a system of Grades, with a nomenclature expressive of the relation of each class to those above, and to those below it. The same is true of the corresponding Abstractions.

The names ‘genus’ and ‘species’ express a single step of the gradation.

The class ‘man’ has a certain degree of generality; it is co-extensive with the human race, and connotes or comprehends the points of similarity among human beings, the terms of communion for admission to the class. The class ‘animal’ is still wider; including human beings and a great many *other* members *besides*—the whole of what is termed the ‘brutes.’ The wider class is called ‘genus,’ with reference to the narrower, the ‘species.’ But there are classes wider still; ‘organized beings’ comprise animals and plants; and if this

wider class were termed a 'genus,' animals and plants would be species under it. The yet higher genus 'material bodies,' would have, as species, organized bodies and inorganic substances; and so on.

Justice is included in the wider class 'virtue;' virtue in the still wider, 'human conduct.' 'Reason' is a species in the genus 'intellectual power;' which last is a species in the higher genus 'mental endowment.'

Circle is a species in the genus 'curve line.'

Geometry is a species in the genus Mathematics; Mathematics is a species in the still higher genus 'science.'

If we had no other terms of gradation but the two—genus and species—obtained from Greek philosophy, we should have to keep shifting them up and down the scale; and they would express nothing but the relationship of the two classes indicated; the genus would always be wider or more general than the species. But in Natural History, where there is a great range of successive gradations, a series of terms has been adopted to correspond to the entire compass of the scale, and each is retained for a distinct grade; 'genus' and 'species' being fixed at a certain stage, and kept always the same. Man, horse, dog, cat, are Species, and are never anything else; the grades next above them are Genera and nothing else.

In Botany, for example, there are four permanent leading grades,—CLASSES, FAMILIES or NATURAL ORDERS, GENERA, and SPECIES. The *Dicotyledons* are a Class; *Ranunculaceæ*, is a Family or Natural Order; *Anemone* a genus; *Anemone nemorosa* (wood anemone), a species. In particular cases, intermediate grades are inserted. Classes are divided into *sub-classes*; Natural Orders, are divided and sub-divided successively into *Sub-orders*, *Tribes*, *Sub-tribes*, *Divisions*, *Sub-divisions*; genera into *Sub-genera*, *Sections*, *Sub-sections*; Species may have under them *Varieties*. The carrying out of these sub-divisions to the full would make fourteen grades.

In Zoology, the primary divisions or *sub-kingdoms*, Vertebrata, Mollusca, &c., are sub-divided into CLASSES (as Mammalia), SUB-CLASSES (Monodelphia), ORDERS (Primates), SUB-ORDERS (Simiadae), GENERA (Ape), SPECIES (Chimpanzee).

Beyond the Natural History departments, and one or two other exact sciences of classification, as Diseases, the terms 'genus' and 'species' retain their mobile character. In Law, crime would be a 'genus' to the particular kinds of crime—treason, murder, manslaughter, theft, libel, perjury, &c. 'Right' is a genus to the several kinds of right; it is a species

under the higher genus 'claim,' or requisition. (G. C. Lewis, 'Explanation of Political Terms,' p. 7).

6. On the principle of Relativity, every class has its CORRELATIVE class or classes; every real notion has a co-relative notion, also real.

'Little more needs to be said on this head. The principle of Relativity, if true at all, must be true without reservation or exception. We cannot form a class, without dividing the universe into two halves, one half within and one half without; when we indicate the class 'round' in the universe 'plane figure,' we imply certain other figures, as triangular, oval, spiral, &c., which are the correlative group. The class 'virtue' supposes another class, according to the universe of the speaker; if that universe be actions relating to morality or to good and evil, the negative or co-relative class is 'vice.' If plants be spoken of, the class to be excluded or denied, may be animals, or may be all material bodies. The class 'bitter tastes,' if in the universe 'sensations of taste,' co-relates with 'sweet, astringent,' &c., or all tastes except bitter; if the universe be 'sensation,' the remaining sensations of taste, and all the sensations of all the remaining senses, are the correlative, the things excluded when 'bitter tastes' are mentioned, the things brought forward when bitter tastes are excluded.

In like manner, every abstract idea must have its correlative or counterpart, which must be a reality if the idea itself is a reality. Length (in the universe 'dimension') is opposed by Breadth and Thickness. If 'justice' be a real notion, there must be a reality corresponding to injustice. 'Affinity' is opposed either to 'neutrality' or to 'repulsion,' or to both. If there be a distinct meaning in 'force,' there must be some distinct opposite; and the meaning changes as the intended opposite changes; it may be force as opposed to inactivity, quiescence, or force as opposed to matter.

THE NOTION UNDER THE GUISE OF THE PROPOSITION.

7. In many instances, propositions appear to give knowledge, but in reality do not; the intention being, not to couple two distinct things in affirmation, but merely to indicate a Class, Notion, or Concept. This is a source of much confusion and fallacy.

In the sentence, 'a triangle is a three-sided figure,' there is

the form but not the reality of predication; in the sentence, 'the pyramid is the form of greatest stability,' there is both the form and the reality. In the first case, what we couple, by the affirmation, is a name and a thing; we give a lesson in naming, or else give the meaning of a name. In the second case, we couple two distinct things; we declare a fact in the order of nature, namely, saying that wherever we find a building of the form of a pyramid, there we have a structure of the highest stability.

The instance first quoted—a triangle is a three-sided figure—typifies a large class of predications in form; they are named 'verbal propositions,' 'definitions,' and also 'analytical' or 'explicative' propositions or judgments. Thus, 'Justice is the giving to every one their due,' is a verbal proposition, definition, or analytic judgment; it tells us, that when the fact—'giving any one their due'—occurs, the single word to name it by is 'justice;' and, conversely, when the word 'justice' is mentioned, the fact signified is otherwise expressed or more fully unfolded by the words 'giving to all their due.' On the one side, such propositions teach us the name to apply to a given thing; on the other side, they teach the meaning of a given name.

In contrast to these propositions in form, the proposition, strictly so called, is a 'real proposition,' an affirmation (or denial) of conjunction, a 'synthetic' or 'ampliative' proposition or judgment, a declaration of the 'order of nature.'

In verbal propositions that assert the concurrence of a name with a single feature of resemblance, there is seldom any mistake. Fallacies do occur in the more difficult and subtle questions; as in Butler's allegations about Conscience and about Right. When persons happen to be very ignorant of a subject, they may fall into the mistake of supposing the declaration of the meaning of a name to be the conjunction of two things, or two facts. Such ignorance is beyond the scope of Logic, which can only give warning of the ambiguous and deceptive character of the propositional form.

'Homer wrote the Iliad,' is a verbal predication. We know nothing about Homer except the authorship of the Iliad. We have not a meaning to attach to the subject of the proposition, 'Homer,' apart from the predicate, 'wrote the Iliad.' The affirmation is nothing more than that the author of the Iliad was called Homer.

'Instinct is untaught ability' is a verbal proposition. If it imparts information beyond the use of the word instinct, the

information consists in substituting a precise statement of the nature of instinct, for a vague and confused one. All improvements in the defining of words have the same effect; and may, therefore, do more than communicate a lesson in naming. This follows from the high function of a general name, which assimilates and brings together widely distributed particulars.

'Instinct is hereditary experience' (Darwin and Spencer), is a *real* proposition; the predicate is an entirely new fact, nowise comprised under the subject.

'Conscience possesses authority over men's actions,' is a verbal proposition. When we enquire into the meaning, connotation, or definition of Conscience, we find that authority is its essential fact; take away authority, and conscience would no longer be present. There may be many *real* affirmations respecting Conscience. We may declare it to be—a simple faculty of the mind, a compound or derived faculty, the vicergerent of the Deity in the human mind, present in all men, absent in some men, absent in the animals, essential to human society, the highest dignity of man.

'Matter is inert' is a verbal proposition; it only repeats the essential quality of material bodies. Real propositions respecting matter would be such as these—Matter is, or is not, eternal; is indestructible; is never at rest; is of many different species; gravitates; is endowed with numerous attractions and repulsions.

'Governments are not made, but grow' is real.

'Justice is honourable,' 'virtue is lovely,' are real propositions, on the supposition that we do not include approving sentiment in our ideas of those qualities.

'Uninteresting sensations are never, for their own sakes, an object of attention,' is a verbal proposition. The predicate 'being an object of attention' means the same thing as the subject 'uninteresting sensations.' To interest us and to excite our attention have scarcely an assignable shade of difference; although it may happen that the use of the designation in the predicate may assist a person little informed to see the full force of the designation in the subject.

'Sovereignty is the authority of one or more men over others' may be given as the meaning of the word, and is therefore a verbal predication. All hypotheses as to the actual, or the legitimate, *origin* of the sovereign power, are real predications.

8. When a class has *several* attributes in common, there

may be the semblance of real predication, yet without the reality.

'A house is made to dwell in' is not a real proposition. 'To dwell in' is a part, although not the whole, of the meaning of a house. Whoever knows what a house is, knows the fact asserted in the proposition.

'Mind is intelligent' is a verbal proposition; the predicate repeats what is already included in the subject. The connotation, or meaning of mind, embraces Intellect, together with two other functions—Feeling and Will. On the other hand, 'Mind is coupled with a material organization' is real; the predicate is no part of the meaning of the subject. We do not include the material accompaniment in the explanation of the word 'Mind.' Aristotle did include, in the meaning of 'soul' $\psi\upsilon\chi\acute{\eta}$, the bodily organization; to him, therefore, 'Soul is coupled with body' was a verbal or analytic proposition.

'Fire burns' is not a real proposition; it merely repeats, or unfolds, the chief attribute of the subject. Our earliest, and most persistent notion of fire, is the same as is expressed by 'burning.'

9. In the Natural Kinds, verbal predication is still more apt to be confounded with real.

A natural kind is distinguished by containing not one, two, three, or four features of community, but a very large, indefinite, and perhaps inexhaustible number—twenty, fifty, or a hundred. Oxygen has a great many properties; the aggregate of all these is properly the meaning of the word. Oxygen is a gas, has a given atomic weight, combines with hydrogen, &c.,—are all in strictness, verbal or analytic propositions. Are they therefore useless or incompetent? Certainly not, yet their form is somewhat misleading.

The technically correct form of these predications would be as follows:—There exists in nature an aggregate of the following properties:—matter, transparency, the gaseous form, a certain specific gravity, active combining power, and so on;—to which aggregation is applied the name 'oxygen.' After the information thus given is fully imbibed by the hearer, the propositions 'oxygen is a gas,' 'is an active combining agent,' &c., are verbal, identical, or tautological propositions; the predicates, being suggested to the mind when the name is pronounced, are, a superfluity.

There are, however, certain circumstances and occasions when such predications are not identical or tautological, but real; the predicate adding something to the subject as understood by the hearer.

(1.) A person may be *insufficiently informed* as to the properties of a certain complex class, but yet may know enough to distinguish the class. Most people know that an elephant is a huge animal, with thick skin, a trunk, and ivory tusks. In such a state of knowledge, the affirmation of any one of these facts would be a verbal or identical proposition; it would merely repeat one of the facts already entering into the meaning of the word. But the elephant has a great many peculiarities besides; and the communication of any of these would be real knowledge; they would be 'synthetic' affirmative—statements added to what is already implied by the word. Yet after being communicated, understood, and impressed in the memory, they would cease to be real predications; they would henceforth be verbal or analytic statements; repeating what the name now suggests or connotes to the person whose information has been enlarged.

All newly discovered properties are real predications on their first announcement; although immediately on being communicated, they become verbal. When Faraday discovered that oxygen is magnetic, the intimation of the fact was for the moment a real proposition respecting 'oxygen'. After being once communicated, it was no more real than the affirmation of any other property of oxygen.

(2.) There may be an *inductive operation* required to ascertain the fact that the properties of a complex class or notion do actually go together in nature. Thus, Mind is defined by the three facts—Feeling, Will, and Thought;—but this supposes a foregone induction, to show that these three properties always concur—that where there is Feeling, there is also Will, and where there is Will, there is also Thought. To affirm that Feeling, Will, and Thought are associated, is a real proposition. The definition of Mind tacitly assumes that this conjunction is established; hence Mind feels, Mind wills, Mind thinks, are verbal propositions. Yet, since they imply, when taken together, that the three distinct facts are united in nature, they may be considered as having the reality of predication underneath.

In like manner, the affirmations—'Chemical affinity is in definite proportions, produces heat, is followed by change of properties'—are a series of verbal or analytic affirmations

yet, there is a reality at bottom, namely, that 'union in definite proportions is conjoined with evolution of heat and change of properties.' The name 'chemical affinity' covers all three facts; and when used as a subject, with any of them as predicates, the affirmation is strictly verbal or identical; the word already means what is affirmed.*

The cases now quoted differ essentially from the aggregates called 'kinds'—mineral, vegetable, and animal bodies, for reasons to be afterwards given.

(3) The verbal proposition may be not improperly used as a *reminder*, or by way of referring to, or reciting a known fact. We may say 'oxygen is the supporter of combustion,' intending only to bring to mind or to indicate that special property with a view of making some inference from it. It is as if we were to say—'inasmuch as among the aggregate of powers and properties named oxygen, one is the support of combustion, therefore, &c'

10. The verbal proposition is, to a great extent, identical with the Definition, which has the form of predication, but is in substance coincident with the Class, Notion, or Concept.

In defining, we use the form of the proposition;—'a square is a straight-lined, four-sided figure, with its sides equal, and its angles right angles;' 'a society is an aggregate of human beings under a common government.' But the alliance indicated by the affirmation is not between two things, but between a name and a thing, so that all definitions are verbal propositions; and all verbal propositions, relating to general words, serve the ends of the definition. The examples above given of the verbal proposition admit of being expressed as definitions, in whole or in part. 'Matter is inert' may be given as the definition of matter. 'Oxygen is a gas,' is part of the definition of oxygen.

11. The Definition, in its full import, is the sum of all the properties connoted by the name. It exhausts the meaning of a word.

* Many words, from the circumstance of naming complex notions, covertly affirm propositions; they cause it to be supposed that the conjunction of the several properties has been already verified; which may or may not have been the case. The name 'substance' means a self-subsisting entity, underlying and supporting the attributes of things, it being taken for granted that there is in nature such a conjunction. Bentham described certain names as 'question-begging appellatives,' because they could not be used without assuming the truth of propositions.

The definition of 'Wealth' is a statement of everything involved in the meaning of the word. The definition of 'Mind' exhausts the properties requisite to whatever we call a mind.

12. When a thing has numerous properties, as in the case of a natural Kind, certain purposes may be served by an unexhaustive definition.

(1.) Instead of our enumerating all the properties essential to a kind, we may mention only those that are sufficient for discriminating it from other kinds. Thus gold could be defined as yellow, incorrosible, and having the specific gravity 19·34; there being no other substance possessing the same combination of qualities. Mercury is the metal that is liquid at common temperatures. The banyan tree sends down numerous shoots which take root and prop up its branches. The elephant could be defined by his trunk alone; this would be quite enough to prevent his being confounded with any other animal. Man could be defined by the number of his muscles, the structure of his hand, or his mental faculties, all which are peculiar to humanity.

These are the definitions that serve for discrimination, testing, or diagnosis. Weight and colour together are sufficient to detect a bad sovereign. In chemical testing, two or three properties are sufficient to identify a substance. There are diseases known by a single symptom; the deposition of urate of soda happens only in gout.

The sufficiency of such definitions is owing to the absence of other things possessing the same features. New discoveries may take away this advantage. The high specific gravity and the colour of platinum failed as decisive tests when the allied metals, osmium and iridium, were brought to light. If there were quadrupeds possessing the mental faculties of man, these faculties would no longer suffice to identify a human being.

(2.) Such definitions, although unexhaustive or incomplete, are yet essentials of the thing defined; they are included among the marks or characters believed to be inherent in the thing. There may be other characters, serving the purpose of discrimination, that are *accidents* and not essentials. Thus, it is an accident of the diamond to be, quantity for quantity, the most precious substance in nature. It is the accident of man to be 'the paragon of animals;' what we regard as the essential features of humanity would still remain, although a higher creature were to appear on the earth. Now, so long as these accidents are distinctive, they serve for a definition, in

the sense of a test; they prevent the thing from being confounded with any other thing known at the time.

If we know a thing *only* by such *discriminative* tests, the other properties, when predicated of it, make, not verbal but, real affirmations. Yet, as soon as we learn these additional properties, we must regard them as falling under the connotation of the word. When we are told that diamond, which we knew to be a transparent, glittering, hard, and high-priced substance, is composed of carbon and is combustible, we must put these additional properties on the same level as the rest; to us they are henceforth connoted by the name.

THE FIVE PREDICABLES.

13. The Five Predicables relate to the distinction between verbal and real predication. They are Genus, (γένος), Species, (εἶδος), Difference, (διαφορά), Property (ἴδιον) Accident or Concomitant (συμβεβηκός).

The three last—DIFFERENCE, PROPERTY, and CONCOMITANT—are the predicates strictly so called, as illustrating the distinction above mentioned. The two first—*genus* and *species*—have nothing to do with predication in the sense of the others.

Genus, Species, and Difference are mutually correlated; each involves the two others. We have already given the meanings of Genus and Species; we have now to add the meaning of DIFFERENCE, which is involved in these. The Difference expresses *the characters possessed by any species, over and above the characters of the genus*. If we suppose 'wolf' to be of the genus *canis*, the characters belonging to the wolf, in addition to those of the genus, are the Difference, Differentia, or specific difference of the wolf. In short, the surplus of connotation of the species, as compared with the genus, is the Difference.

'Science' being called a genus and 'chemistry' a species under it, the *differentia* of chemistry is what distinguishes it from other sciences, what it has peculiar to itself, besides the generic features of a science.

Of the three facts—genus, species, difference—given two we infer the third. From the genus and the species, we can tell the difference; we have only to subtract the essential attributes of the genus from the essential attributes of the species. Given the species and the difference, we can find the genus by subtracting the difference from the attributes of the species.

Given the genus and the difference we can fix the species, by adding the generic marks to the difference. Fine Art being a genus, and Painting a species, the difference is the medium or instrumentality of colour.

14. A short, and yet complete, form of Definition is to state some higher genus of the thing defined, together with the specific difference. In popular language, defining often assumes this form, and it has been improperly regarded by logicians as the regular and only form.

Physiology is defined the *Science* (genus) that treats of *living or organized bodies* (difference). Poetry is a *Fine Art* (genus) having *language for its instrument* (difference).

Ordinary speech being addressed to persons already partially informed, it is usually sufficient to define in this way. The person wishing a definition of Physiology is supposed to be already familiar with the generic idea of science. If this is not the case, the definition fails. Science itself would require definition by reference to a higher genus as 'knowledge,' and so on.

15. All the attributes of the genus, and the additional attributes of the species (that is, the difference) are considered to be ESSENTIAL attributes. They are all included in the meaning or connotation of the name. Hence the affirmation of these makes a verbal (or *essential*) predication.

The generic characters of 'canis' and the additional or specific characters of the wolf are, by the very nature of the case, the characters connoted by the terms 'canis' and 'wolf.' To say otherwise would apparently be a contradiction in terms. But the force of the remark is not brought out until we advert to the two remaining heads of predication,—Property and Concomitant.

16. Property, or PROPRIUM, belongs to *real* predication. It means an attribute flowing out of, deduced from, or dependent on, an essential character.

The meaning, connotation, essence, or definition of a triangle is a right-lined plane figure with three sides. There follow from this definition, by geometrical deduction, a great many propositions relating to the triangle;—as 'any two sides are greater than the third,' 'the three angles are equal to two right angles.' These fall under the head of predication called 'pro-

perty' or *proprium*; they are not essential characters, although derived from essential characters. They typify one large department of real predication—the propositions obtained by *mathematical* inference.

Again, 'oxygen supports combustion' is not an essential quality of oxygen; it is a *proprium*. It is clearly deducible from the more general quality of oxygen expressed by its combining powers: it is more immediately derived from the fact that oxygen combines with carbon.

From the specific gravities of a number of substances (an essential quality), we can deduce a great many *propria*. Comparing, on the point of specific gravity, mercury with platinum and gold, we infer that platinum and gold will sink in mercury; a similar comparison would show that iron, tin, copper, lead, silver, &c., will float. These are deduced propositions or *propria*, and not essences; they are not generic, specific, or differential characters.

'Fluids press equally in all directions' is a *proprium*; it follows from the definition of fluidity.

The power of speech is not an essential or defining character of man; it proceeds from his other endowments of body and of mind; it is a *proprium*.

We see, therefore, that to keep up the distinction of essence and property, it is requisite that the essential or defining marks of a thing should be *ultimate* and distinct, and not resolvable into one another. If a quality could be shown to flow from some other quality, it would cease to be an essential or defining mark, it would be an inference or *proprium*. The distinction is lost, when we mix up indiscriminately ultimate characters with derived characters, as is not unfrequently done in the sciences, as well as in popular usage. The enumeration of the attributes of oxygen, of gold, of man, should be an enumeration of the final (so far as can be made out), the underivable powers or functions of each.

The proposition 'Man is rational' is a *proprium*. The ultimate analysis of man's mental nature, to which 'rationality' is referable, shows that reason is not a fundamental operation, but derived from the foundations of the intelligence; whence this should not be given as part of a scientific definition of man.

The same may be said of 'Man walks upright'; which is an easy inference from his anatomical structure. So also 'man is a cooking animal,' would be an application of a more general fact—man is a tool-using animal; which is itself a

derivative from his muscular endowment combined with his intelligence.

The proposition 'man is mortal' is expressly given by Mr. Mill to exemplify real, as opposed to verbal, predication. If so, it is a *proprium*. To decide the question, however, we should have to go back to the mode of stating the peculiar feature of organized beings that refers to their germination, growth, and decay. Should the cycle of existence signified by these words be reckoned an ultimate, or unanalyzable attribute of living beings, mortality would be of the essence of men, as of all animals, and all plants; and therefore to affirm it would be a verbal or essential predication.

17. The ACCIDENT or CONCOMITANT, in Predication, expresses something neither belonging to the essence or connotation of the subject, nor deducible from it. 'Gold is the most valuable of the metals,' 'is used for the coin of the realm'—are propositions where, the predicate would be called an Accident or Concomitant.

The real proposition, as opposed to the verbal, essential, or identical (Kant's *analytic* judgment), reaches its highest point, in this species of predication. It gives us the full meaning of Kant's '*synthetic* judgment,' where the predicate is a positive addition to the subject, and neither directly nor indirectly contained under it.

These affirmations of concomitance are exceedingly abundant in everyday practice. We are constantly finding about us things joined together, without mutual implication. All the affirmations respecting material bodies that deal with their local distribution, their quantity, their uses,—are affirmations of concomitance; we do not include these points in the definition or essence. It is the essence of gold to be incorrosible (unless it were to be found to be derivative, or a *proprium*); it is not the essence to be used for coin, or for ornament; still less is its occurring in California and in Australia. We should not think of including these facts in the definition of gold. The specific gravity is an essential quality (to all appearance); and doubtless the position in the older and deeper rocks is a consequence of this, and might be called a *proprium* of gold.

The putting forth of energies into actual display is the occasion of propositions of concomitance. Socrates sits, walks, converses, are real predications. All the shifting usages, habits, and positions of things, are in like manner real:—he is in good

health; the mountain is covered with snow; the crops are ripe.

Among the highest propositions of science, as will be seen afterwards, there are few predications of concomitance.

18. A distinction is made between *separable* and *inseparable* Concomitants. The inseparable Concomitant is scarcely distinguishable from the Essence.

The separable concomitant is what we commonly mean by Accident; as 'gold is found in California.' We see plainly that this depends upon arrangements where other matters besides gold are concerned; and which might have been different without any alteration in the qualities of gold itself. That geese were kept in the capitol of Rome, was an accident, a separable concomitant, of the goose.

The standing example of this distinction in the old logical books was 'Virgil resides in Rome' (separable), 'Virgil was born in Mantua' (inseparable); a distinction sufficiently real, but practically worthless.

The inseparable concomitant is exemplified in the colour of those animals whose colour has never varied; as was so supposed to be the case with the whiteness of the swan and the blackness of the crow. If we were to ask why an attribute always present in a species, and not known to be a *proprium*, was not adopted into the Essence, we should probably be told in reply, that the colour of animals is an unstable property; it often varies when everything else seems to remain the same; hence it is usually left open in assigning the marks of species. The cases quoted justify the practice. Neither the whiteness of the swan, nor the blackness of the crow is universal in those species.

These remarks on the Predicables will serve to bring out into farther prominence, the distinction between Verbal and Real predication.

CHAPTER III.

PROPOSITIONS.

1. The Proposition has been already viewed as made up of Subject, Predicate, and Copula.

In common with names, and with notions, Propositions may be classified (I.) according to *Generality*, and (II.) according to *Relativity*.

We now enter upon the full consideration of the *Real* Proposition, where there is both the appearance and the reality of predication.

It is of importance to view propositions, as we have viewed names and concepts, with reference to the two fundamental attributes of knowledge—Agreement and Difference, or Generality and Relativity.

I. Propositions follow concepts in being of different grades of GENERALITY. '*The St. Lawrence falls at Niagara*;' '*all water descends*;' '*all terrestrial bodies gravitate towards the earth's centre*;' '*the bodies of the solar system gravitate towards each other*;' '*all matter gravitates*;'—are propositions of successive degrees of generality; each takes a wider sweep than the previous, till we reach the widest of all. '*People should be taught not to take cold*'—'*to take care of health*,' '*to be prudent*,' '*to be virtuous*,'—are four propositions rising in generality.

It is obvious that the generality of the Proposition follows the generality of the concept or notion. Any proposition respecting the Earth, is merged in a proposition respecting the planets; a proposition respecting the Planets is less general than one respecting Heavenly Bodies. The more general the concept forming the subject of a proposition, the more general the proposition: '*men, animals, organized beings*,—are liable to disease.'

The law of inverse relationship of Extension and Comprehension—Denotation and Connotation, applying to the notion, applies also to the proposition. The most highly generalized propositions are those that have the smallest predication; the extent is gradually lessened as predication is increased, We

say '*all matter* is indestructible,' but when to the property of indestructibility we add the property—unchangeable in state (as regards solid, liquid, gas)—we have to limit the subject to a few bodies, as to the (hitherto) *uncondensable gases* and to *carbon**

II. Propositions come under RELATIVITY, in this respect, namely, that to every proposition there exists a correlative proposition, something denied when it is affirmed. 'Europe lies north of the Equator'—'Europe does not lie south of the Equator'; 'friendship is pleasure'—'friendship is not painful nor indifferent.'

Here, too, the proposition follows the notion. To every intelligible notion, there is an intelligible opposite—something that remains when the notion is subtracted from the universe; south is opposed by north (universe 'north and south'); plea-

* The circumscription of general maxims, with reference to actual cases of practice, is thus effected by *adding* the circumstances of the given case, and considering the combined result. A general theorem is founded on a limited set of hypothetical data, and the more limited they are the more abstract is the theorem. The intensity varies inversely with the extent of its signification. Now a theoretical proposition, when converted into a rule of conduct, may be conceived as taken in connexion with an indefinite number of sets of concomitant circumstances, which may modify its operation. If, therefore, we add a definite number of circumstances to the proposition, we exclude all uncertainty as to the possible combinations, and we in fact perform a sort of practical *abscisso infiniti*. We substitute a real and definite for an ideal and indefinite compound. The addition of a limited number of terms operates as the exclusion of an unlimited number.

Thus, let it be supposed that our general theorem is as to the operation of legal punishment. Legal punishment, if left to itself, may be expected to produce abstinence from crime; but it may be accompanied, and as it were, held in solution by a vast variety of collateral circumstances which may influence its operation. Thus, it may be combined with an inefficient or unskilful police, a venal, or tardy administration of justice, difficulty of detection, unwillingness to prosecute or to give evidence, or a fanatical contempt of suffering. Various other circumstances might likewise be mentioned which diminish the deterring force of the fear of legal punishments on the minds of given individuals. Now, all that can be said with reference to such a general theorem, so long as it remains an abstraction, is that it describes a prevailing tendency, liable to be resisted and modified by an unlimited number of counter-influences with which legal punishment may be combined. But when an actual case is laid before us, we can perceive whether any is, and which of those other circumstances are present. Of such as are wanting we take no account, we note those which are discernible, and we then form a definite practical problem, in this shape: 'How will the denunciation of legal punishment operate, taken in connexion with a reluctance of witnesses to give evidence, or with a willingness of judges to take bribes (as the case may be)?' 'What will be the effect of legal punishment, combined with a hope of impunity, or a disregard of pain, of some special ascertained nature?' (G. C. Lewis).

sure is opposed by the two states—pain and indifference (universe ‘feeling’).

These two fundamental distinctions, applicable alike to the Notion and to the Proposition, being presupposed, we proceed to the various classes of Real Propositions that have a bearing on Logic. The primary division is according to EXTERNAL FORM, and according to IMPORT, or Meaning, in the final analysis.

The term ‘Judgment’ is used in most logical treatises to designate the proposition. A proposition is stated to be ‘a judgment expressed in words;’ and Judgment is termed the mental operation whereby we pronounce two things to agree or disagree. When we affirm a mountain to be four thousand feet high, we pronounce the agreement of the height of the mountain with the lineal quantity denominated four thousand feet; we of course imply the disagreement with any other quantity more or less.

It may be remarked on this employment of the word Judgment, in connexion with the proposition, that, in the view of Aristotle, it had a real significance. Aristotle took account of the *subjective* element of affirmation, the implication of the individual mind of the affirmer in the process. When I say, ‘the earth is round,’ the full import is that, according to my belief, conviction, or judgment, the earth is round; or I believe that the earth is round. I speak only for myself. I cannot undertake to say what other people believe, unless they tell me; and, apart from all belief, the proposition has no meaning, no existence.

For almost all practical purposes, this indispensable correlate may be left in a tacit condition. Being always presumed, it need not be mentioned. In many other cases, we suppress the mention of what we can always count upon, as for example, gravity. We do not say that a certain weight will maintain the movement of a clock, *provided gravity continue to operate*; we take this for granted without specifying it. But there are occasions when the correlated subject in affirmation needs to be brought into view; as when metaphysicians declare that there can be objective truth without a subject; and when certain opinions are sought to be imposed by force, as absolute and infallible.

Apart from this circumstance, the term Judgment is not the most apposite word for expressing the formation of propositions. The function of a judge may require propositions to be stated; but more usually it consists in discerning the agreement or disagreement of a proposition with a given case; as in the interpretation of the law. The faculties needed for arriving at propositions are much more extensive than is meant by judgment; the processes of observation, classification, induction and deduction, bring into play the senses and the intellectual powers in their widest scope.

It is incorrect and misleading to describe a proposition as ‘judging two notions to be congruent,’ ‘conceiving them as one’

(Hamilton). All that a proposition can do is to *link* together two facts (as 'fluid' and 'level'), it does not in any sense make them one fact, or bring the one under the other.

EXTERNAL FORM OF PROPOSITIONS.

- 2. Propositions are either *Total* or *Partial*, which distinction is expressed by the word QUANTITY.

Universal and *Particular* are the names most used, although not the aptest, for signifying this division.

When the predicate is true of the subject, in its whole extent, or in every instance, the proposition is *total* or *universal* in quantity;—'*all* the planets are round'; '*all* the virtues are useful'; '*all* coal is the product of ancient vegetation'.

When the predicate is true of the subject, only in part of its extent, or in an indefinite number of instances, it is *partial* or *particular* in quantity:—'*Some* planets are larger than the earth'; '*some* of the virtues are painful in the performance'; '*some* coal is useful for making coal gas'; '*some* men are wise'; '*some* metals are incorrosible'; '*some* crystals are transparent'; '*some* diseases are incurable.'

The usual designations for total or universal quantity are 'All' and 'Every.' 'All earths are oxides of the metals'; 'every man is expected to do his duty.' There is a rhetorical, but no logical distinction, between the two; 'every' has the emphasis of greater individuality. 'All' is sometimes ambiguous; it may be used in a collective, as well as in a distributive sense; 'all England' may mean the whole nation in a collective capacity, and not 'every Englishman.'

Universal quantity is sometimes given in less explicit forms:—'*The earths are oxides*,' '*evil-doers need to be punished*,' '*man is frail*,' '*pleasure tempts*,' '*alcohol is a stimulant*,'—are understood to be universal, although they have neither the decisiveness nor the emphasis of the other forms.

The term of Partial or Particular quantity is '*some*,'—meaning an indefinite number, one or more, and possibly all. It negatives '*none*,' without saying how many. The logical '*some*' is expressed by the phrase '*some at least*.' The '*some*' of common speech is different; '*some men are wise*,' '*some fever-patients recover*,' are interpreted as implying that there are some men that are not wise, and some fever patients that will not recover. When we assert a quality of a subject that we are acquainted with, we are usually aware that while some instances possess the quality, others do not; the use of '*some*'

does not express our ignorance of the others, but rather our knowledge that these are deficient in the quality. This is fully stated by 'some at most,' a small or limited number, in comparison with the whole. The logician's view of 'some' would correspond to a case of first contact or encounter with a new class of things. Thus, a voyager in landing on a newly discovered coast, and meeting a few of the inhabitants, while as yet ignorant of the general mass, would say 'some are lank-haired;' he would speak of those he saw, and of no more.

The logician's 'some' is rarely found in common use. The word itself is frequent enough; but in using it, we are aware that there is an actual limitation of the subject. The logical importance of the word comes out in the *conversion* of propositions, with a view to the syllogism. As, in nearly every affirmative proposition, the predicate is larger than the subject, includes the subject and something more, we can never transpose the terms (in conversion) without a qualification; 'all men are mortal,' if transposed, must be 'some mortals are men.'

In what is called the 'minor term' of the syllogism, 'some' can be replaced by any other word of quantity, as one, ten, few, a small number, many, &c.; the same word being transferred to the conclusion keeps the syllogism correct. But in the really important case—the expression of a universal affirmative, in *transposed terms*, we are restricted to 'some' or 'part.'

The reason why 'Universal' and 'Particular' are not suitable names, for the two modes of quantity, is that these names designate also the inductive contrast between a general proposition and the particulars or individuals that we derive it from. The distinction of General and Individual belongs to the substance and not to the form of propositions; it is their *inductive* and not their deductive, or formal aspect.

Mr. De Morgan (Syllabus, p. 60) proposes the terms 'full' and 'vague' as other synonyms for the objectionable couple—Universal and Particular. 'All Men' is *full extent*; 'some men' is *vague extent*.

Another term for quantity less than total, is 'Most;' which has been introduced into the syllogism by Mr. De Morgan; 'most gases are odorous:' 'most of the cerebral nerves spring from the medulla oblongata;' 'most plants are hermaphrodites.'

Certain forms of the proposition have been called *Indefinite* in quantity; the expression leaving it uncertain whether they are universal or particular. They are, in point of fact, *ambiguous*. The chief examples occur with names of material,

which are the subjects, sometimes of universal, and at other times of particular, predication. 'Food is chemically constituted by carbon, oxygen, &c.' is a proposition of universal quantity; the meaning is all food, all kinds of food. 'Food is necessary to animal life' is a case of particular quantity; the meaning is some sort of food, not necessarily all sorts. 'Metal is requisite in order to strength' does not mean all kinds of metal collectively. 'Gold will make a way' means a portion of gold.

The term 'Distribution' or 'Distributed' is a technical, but not very suggestive, term for universal quantity. With the universal designations 'all,' 'every,' or their equivalents, a subject or predicate is said to be distributed; a particular form 'some' is said to be undistributed.

3. Propositions are either Affirmative or Negative; a distinction according to QUALITY.

A proposition either affirms or denies a predicate of a subject; 'Wine is good,' 'wine is not good.' Two properties either co-exist or do not co-exist; and to be informed of non-co-existence is as important as to be informed of existence. 'The moon is up,' 'the moon is not up,' are propositions equally valuable as knowledge; we are guided by the one no less than by the other. 'He is guilty,' 'he is not guilty,' are fundamentally different assertions; each drawing its own consequences with it.

Affirmative and Negative propositions are not merely different, they are *opposed*; which signifies that by interpreting the opposition, we can make out all the consequences of the one from the consequences of the other. With the same subject and the same predicate, affirmation and denial are so implicated together, that if we know what the affirmation means, we also know what the denial means. One effort of understanding serves for both. If we are told that 'the accused is guilty,' involves a fine of five pounds; we know also that the negative, 'the accused is not guilty' involves exemption from the fine. This is merely an aspect of the Law of Relativity; according to which the knowledge of opposites is one.

Some logicians have proposed to do away with the distinction between affirmative and negative by transferring the sign of negation from the copula to the predicate; 'A is not B,' 'A is not-B;' 'penury is not-agreeable,' 'penury is disagreeable.' There is then the appearance, but only the appearance, of making all propositions affirmative. The attempt is illusory. Affirmation and Denial belong to the very nature of things; and the distinction, instead of being concealed or disguised to make an imaginary

unity, should receive the utmost prominence that the forms of language can bestow.

Thus, besides being either universal or partial in quantity, a proposition is either affirmative or negative. And, by the Law of Relativity, to every affirmative form there corresponds a negative form, both understood if one is.

Negation is complicated by the quantity of the propositions opposed. The simplest form is seen in the opposition of a universal to a universal—‘all diamonds are precious,’ ‘no diamonds are precious,’ or when the subject is a definite individual, as ‘Francis was (or was not) the author of Junius.’ When a particular is opposed either to a universal, or to another particular, there arise distinct forms of negation or contrariety, which will be described presently.

4. The negative words ‘not,’ ‘no,’ and their equivalent prefixes and suffixes, are the explicit forms of negation. There are other forms of a less direct kind,

For the negative of a definite particular proposition, as ‘John is here,’ ‘the day is fine,’ we prefix *not* to the predicate, ‘John is *not* here.’ For universal propositions, this mode is insufficient; ‘all planets are round,’ is not negated by ‘all the planets are *not* round;’ the meaning of such an expression, according to the idiom of our language, is, that some planets may be (and probably are) round, but a reservation is made of the rest. We arrive at a thorough negation, to the complete denial of the universal affirmation, by prefixing the negative adjective ‘no’ to the subject—‘no planets are round.’

‘No useless coffin enclosed his breast;’

Another form, adopted for rhetorical emphasis, is seen in ‘*not a man* escaped,’

‘Not a drum was heard, not a funeral note.’

The prefixes ‘in,’ ‘un,’ and the suffix ‘less,’ are equally emphatic. ‘All his actions were just (unjust), wise (unwise), prudent (imprudent).’ ‘The country in stony Arabia is waterless and treeless.’

Negation may be conveyed by such phrases as ‘far from,’ ‘the reverse of,’ ‘on the contrary,’ ‘wanting or deficient in,’ ‘devoid of,’ &c. Certain words, as ‘few,’ ‘hardly,’ ‘scarcely,’ have a positive or negative effect according to the context. ‘Few’ admits a small number, and denies all beyond; occasionally it is a polite form of total denial. In some cases, the meaning is positive, the stress being laid upon the small

amount of admission ; in other cases, the force is meant to be negative, 'few will see that day.'

5. Propositions are either SIMPLE or COMPLEX, a distinction only partially belonging to Logic.

• In a simple proposition, there is but one subject and one predicate : 'the sun is up,' 'justice is excellent,' 'Britain has numerous colonies.' In a complex proposition there are more than one predicate or more than one subject, or both. 'Britain, France, and Prussia are maritime powers;' 'Britain has often been at war, and has acquired foreign possessions.' In the first example, three propositions are combined in one common predicate ; and should they require to be logically canvassed, they must be taken separately : 'Britain is a maritime power,' &c. In the second example, two propositions are affirmed, and one implied, although there is but one subject 'Britain.' It is affirmed (1) that Britain has often been at war, (2) that Britain has acquired possessions abroad ; and the close connexion of the two statements, is meant to convey an additional circumstance, namely, that the second fact was the consequence of the first. As before, these allegations would be taken in their separate and simple form, in any question as to their truth or falsehood, or as to the evidence in their favour.

The whole of this class might be called Compound, instead of complex, Propositions.

6. The Complex Propositions more especially entering into Logic are of two kinds, named *Conditional* and *Disjunctive*. In these, the separate propositions are conjoined in one meaning.

The *Conditional* Proposition is extremely common ; it is a statement with a qualification ; 'if ignorance is bliss, 'tis folly to be wise' ; 'if every one speaks together, the business cannot be done ;' 'unless rain come, the crops will fail.'

This form is also expressed by saying that one statement is the consequence of another ; or that there is an affirmation of the consequence or connexion of two facts ; that where one fact is present the other fact will follow, these facts being expressed in propositions. Thus, 'the consequence of ignorance being bliss is, that it is folly to be wise ;' 'the consequence of every one speaking together, is that no business is done ;' 'the consequence of a want of rain will be a deficiency of the crops.'

In all such cases, it is only a matter of course, that supposing the antecedent present, the consequent is also present.

The *Disjunctive* Proposition expresses an alternative: 'John is either in the house or in the office'; 'granite is either a sedimentary deposit or a product of igneous action'; 'to be or not to be, that is the question.'

'These propositions may be viewed as condensing alternative conditions; 'If John is not in the house, he is in the office; and if he is not in the office, he is in the house.'

Each class is the basis of a distinct species of logical transformations, constituting a supposed variety of the Syllogism. The name 'hypothetical' expresses both the conditional and the disjunctive forms, and is opposed by 'categorical' which designates all other propositions.

7. The combination of difference in Quantity with difference in Quality, gives rise to four classes of Propositions :—

- (1) Universal Affirmative (A)
- (2) Particular Affirmative (I)
- (3) Universal Negative (E)
- (4) Particular Negative (O).

These propositions are expressed symbolically by the letters A, I, E, O. The first and second forms, the affirmative, derive their symbols from the vowels of the word Affirmo: A being the universal, I, the particular affirmative. The third and fourth forms, the negative, derive their symbols from the vowels of nEgO; E being the universal, and O the particular negative.

A—All men are fallible: all X is Y.

I—Some men are wise: some X is Y.

E—No men are gods: no X is Y.

O—Some men are not wise: some X is not Y.

Hamilton's Quantification of the Predicate.

8 These are all the forms admitted into the usual syllogism, being sufficient for ordinary purposes. We may notice, however, in all of them, that the quantity spoken of has reference to the *subject*; and nothing is said explicitly of the quantity of the *predicate*. By supplying this omission, Hamilton has indicated four additional forms.

Thus, to take All X is Y: 'all men are fallible. Y may mean some Y or all Y; some fallible beings or all fallible beings: There are then two forms:—

(1) All the Xs are some (a part) of the Ys; all men are some (a part of) fallible beings. This is what is presumed to be the meaning of the common form, where the quantity of the predicate is not stated. As there is no assurance given that the Xs are all the Ys—that men are the whole of the beings that are fallible—we must leave it to be understood that there are other Ys, other fallible beings, and therefore take for granted only that men are *among* the fallible beings, whether there be others or not. Usually, we do not concern ourselves with this farther enquiry; it is enough for us to know, on a particular occasion, that a certain man, or a number of men, are fallible, or that a certain substance is poisonous, without determining whether others besides those in hand have the same quality. This last is a distinct and superadded enquiry, useful in particular situations, but not in all, nor even in the majority of instances. The fact is valuable to know that 'wines are stimulating or intoxicating,' whether or not they include the whole of the stimulants. It is a farther discovery, having a separate utility, to find that there are stimulants besides wines. The common form is suited to the first case; the quantified form—all wines are some stimulants, there are other stimulants besides wine—is suited to the second case.

On the strict Logical sense of 'some,'—some at least, and it may be all,—the quantified form 'all X is some Y' is the same as the unquantified form 'all X is Y.' There is merely this difference that in the quantified form, attention is called to the circumstance whether there be more Ys than are Xs; in the common form, no question is raised or even suggested as to additional Ys beyond the Xs. If 'some' were interpreted in the more familiar meaning 'some at most,' which it is apt to be, the particular quantification would not give the meaning of the unquantified form.

It will be seen, in the account to be afterwards given of Boole's Logic, that he finds it necessary to express, by a symbol, that the predicate of affirmative propositions is taken only in part of its extent.

(2.) With the predicate made universal, the form A becomes 'All X is all Y;' there are no Ys but the Xs. Such is not a usual form of predication. In the great mass of positive affirmations the predicate is larger than the subject, includes it and other things besides: 'the coin of the realm is metallic;' there are many things made of metal besides coin. 'The stars are heavenly bodies,' but not exclusively so.

To exemplify this kind of proposition, there are offered such

instances as these;—‘Chloride of sodium is common salt,’ which means, there is no chloride of sodium but what is common salt. But these terms are co-extensive only because they are synonymous; they are two names for the same thing. Defining propositions must be co-extensive.

As an example taken from real propositions, we may have this—‘All equilateral triangles are all equiangular triangles;’ for there are none equilateral but are also equiangular. Such cases are not frequent even in the deductions of Geometry, where the propositions affirm *propria*, and not concomitance.

There are a few cases of unique properties furnishing propositions where the subject is as large as the predicate. ‘Mercury is a liquid metal’ is known to be ‘all mercury is all liquid metal.’ In such instances, it is usual to note the fact, that subject and predicate are co-extensive in the language used; as by saying, mercury is the *only* liquid metal; there is no metal liquid at common temperatures but mercury. Being an exceptional predication, it receives exceptional notice. Of a similar nature is Hamilton’s example, ‘All rational is all risible;’ we should say, ‘only rational beings are able to laugh.’

In the more general conjunctions, or concomitance of distinct qualities, it is exceedingly rare to find a proposition where subject and predicate are co-extensive. Only one unequivocal instance can be suggested at the present time, namely, the proposition, ‘all matter gravitates;’ the meaning of which is that the defining property of matter—Inertness—is always accompanied with the attraction of gravitation. Now, these two attributes are co-extensive, and yet distinct; all matter is all gravitating things; there is nothing devoid of inertia, and yet possessing gravity. Even here it may be said, that although we can easily suppose inertia without gravity, we cannot easily suppose gravity without inertia.

Polarization and Double Refraction are co-extensive properties.

Mr. De Morgan, as will be afterwards seen, calls the form a *complex proposition*, being tantamount to two propositions—All X is Y, and all Y is X.

Mr. Mill makes substantially the same criticism on Hamilton’s Quantified forms. Whatever can be proved from “all A is all B,” can be proved in the old form from *one or both of its elements*, All As are Bs, and all Bs are As. ‘Whatever can be proved from “Some, and only some, A is some (or all) B,” can be proved in the old form from its elements, Some As are Bs, some As are not Bs, and (in the case last mentioned) all

Bs are As.' (Mill's Hamilton, chap. XXII). To say 'All Philosophy is all Poetry' is to affirm these two propositions, Poetry is Philosophy, and Philosophy is Poetry.

The Particular Affirmative, I, has two forms, when the quantity of the predicate is supplied:—Some X is some Y, (the understood form), some X is *all* Y: 'Some planets are some celestial bodies;' 'some mortals are *all* men.' The second is the new or additional form. Its best justification is the circumstance that, under the common form, we lose predication in converting a universal affirmative: thus, All X is Y, all men are mortal, become, some Y is X, some mortal beings are men, meaning *some* X, *some* men, whereas we are entitled to say *all* X, *all* men.

These two additional affirmative forms have been admitted by some logicians, as Thomson (Laws of Thought) and Spalding; and have been made the basis of an extension of the syllogism. The universal affirmative—All X is all Y—is symbolized by U (Thomson) and by A³ (Spalding). The particular affirmative with universal predicate is Y (Thomson), I² (Spalding).

The additions made by Hamilton to the negative forms, E, and O, have not been received by any other logician. In E, 'no X is Y,' 'no men are gods,' both subject and predicate are universal; there is total and mutual exclusion; no one of the class men is identical with any one of the class god; the coincidence of a man with a god is denied seriatim. The predicate here is quantified universally. We may, however, state a form where the predicate is particular; 'no X is some Y,' 'no men are some animals,' no men are to be found in a certain class or species of animals; there are classes of animals that entirely exclude men. If the 'some animals' could be specifically defined, as quadrupeds, fishes, &c., the proposition would revert to the common form.

In the Particular Negative, O, 'some X is not Y,' the subject is particular, and the predicate universal. 'Some Xs are not to be found among the Ys;' 'some men are not any Europeans, are not to be found among Europeans;' 'some heavenly bodies do not shine by their own light.'

Now, particular quantity may be assigned to the predicate; which would then be, some X is not some Y; some of the Xs do not occur among some of the Ys. Some men are not to be found among some of the mammals. If 'some of the mammals,' could be rendered specific, as the 'carnivorous quadrupeds,' 'the thick-skinned quadrupeds,' we should have the old

form of O. In answer to the objection against the new form, that it is never practically realized, Hamilton contends that it is the form wherein, exclusively, *we declare a whole of any kind to be divisible*. Thus, in dividing the genus 'soldier,' we should say to ourselves—"some soldier is not some soldier; for some Soldier is (all) Infantry, some Soldier is (all) Cavalry, &c.; and (any) Infantry is not (any) Cavalry."

De Morgan's Enumeration of Propositions.

9. With a view to exhaust all the possible modes of predication, there needs to be a thorough-going expression of *contraries*.

According to the true view of contrariety, as given by De Morgan, the negative is a remainder, gained by the subtraction of the positive from the universe; the negative of X is $U-X$, and may be symbolized by a distinct mark x ; whence X and x are the opposites under a given universe; not- X is x , and not- x is X . For, Some X s are not Y s, we may substitute, Some X s are y s; and so on.

We have now, instead of the two terms X, Y , the four terms X, Y, x, y . Hence, in room of the one couple, X, Y , to be given under the four forms of predication—A, E, I, O—we have no less than four different couples— X, Y ; X, y ; x, Y ; x, y . Every one of these may be stated, as A, as E, as I, or as O. Consequently there are sixteen possible arrangements. On examination, however, eight turn out to be repetitions of the other eight.

We may exhibit the sifting operation thus:—Take A, or universal affirmation, and express all the four couples accordingly.

- (1) All X is Y (the usual form) . .
- (2) All X is y (not- Y)
- (3) All x (not- X) is Y
- (4) All x (not- X) is y (not- Y)

The second—All X is y (not- Y)—is identical with E, in the old scheme—No X is Y .

The third—All x (not- X) is Y , is the same as nq not- X is not- Y ; nothing is both not- X and not- Y ; everything is either X or Y . No not-mind is a not-matter; everything is either mind or matter. This is a *new form*. It means that everything is either in X or in Y (or in both).

The fourth—All x (not- X) is y (not- Y), (all not-mortals are not-men), is the same as All Y is X , a new form, so far, that the symbols are transposed.

Again, putting the four couples through particular affirmation (1) :—

- Some X is Y
- Some X is γ (not-Y)
- Some x (not-X) is Y
- Some x (not-X) is γ (not-Y)

The first being the common form; the second is the common particular negative. The third, 'Some not-X is Y,' may be transformed into 'Some Ys are not Xs,' or 'All Xs are not Some Ys,' in which shape it is received among the additional forms. The last 'Some not-X is not Y;' 'some things are neither Xs nor Ys;' all the opposites of X are opposites of Y. Infantry is neither cavalry nor artillery; the negative of X (cavalry) is the negative of Y (artillery), that is, infantry.

The same method pursued with universal, and with particular negation, completes the survey, and also yields a new form, already quoted,

Some Y is not X

which, like the form—All Y is X—is merely the transposition of the letters in O. The author has special reasons for including these two varieties among propositional forms.

Thus, then, in addition to the old fundamental forms, A, I, E, O, we have these four :—

- (1) Every Y is X
- (2) Some Y is not X

which are A and O, reversing the terms.

- (3) Everything is either X or Y
- (4) Some things are neither X nor Y

These last are a contrary couple of *Disjunctives*, added to the four regular forms, which are all *Categorical*.

The author next adverts to the compatibility or incompatibility of these various forms. There are three alternatives. (1) The separate individuals may be such as *cannot exist together*. (2) They may be such as *must exist together*. (3) They may exist either with or without each other, in *neutral concomitance*. It is evident, for example, with regard to the old forms that A cannot co-exist with E, or with O; if every X is Y, it cannot be true, either that no X is Y; or that some X is not Y. Again, if A exists, I must exist: and so with E, and O; the particular is involved in the universal. Lastly, the particulars I and O, may or may not exist together: they are neutral concomitants; 'some men are wise,' and 'some men are not wise.' [Substantially the statement of the Opposition of Propositions.]

From this, the author proceeds to define what he terms a *complex proposition*; 'one involving within itself the assertion or denial of each and all of the eight simple propositions.' Thus supposing X and Y to be such that none of the four universals are true; then all the four particulars are true. This is one case, called a complex particular. Another case is to suppose one of the universals true; then five others are settled, either by affirmation or by denial: and there are two concomitants, which however, are contradictions, so that only one is true. Of this generic character, there are six modes or forms; one of which has an especial interest.

The case is this, Let A (the old form), 'Every X is Y' be true. Then E and O, are denied, and I, is included (of the old forms). Of the four new forms, the neutral concomitant is 'Every Y is X': these may co-exist, and when taken together make the complex proposition - Every X is Y, and every Y is X: in other words, X and Y are co-existent, or identical. Now this is Hamilton's Universal Affirmative, with universal quantity in the predicate. All Xs are all Ys. So that, in De Morgan's view, that form has no claim to be a simple or fundamental propositional form; it is a compound or complex proposition, derived from the simple forms, by the process now described. He supports this view, by the farther allegation, that the proposition in question does not admit of a simple denial, as every proposition of a fundamental kind should: it is contradicted either by 'Some Xs are not Ys' or by 'some Ys are not Xs'; that is, by the disjunction 'either some Xs are not Ys, or some Ys are not Xs'; and it is not necessary to determine which, so that the contradictory is ambiguous or undecided.

Opposition of Propositions.

10. Negation in the full sense is exhibited by opposing a Universal Affirmative to a Universal Negative—A to E, as 'all men are wise, no men are wise.' This is called, in Logic, the opposition of CONTRARIES.

Contrariety, in this sense, is the setting up of a Universal Negative, against a Universal Affirmative, or a Universal Affirmative against a Universal Negative: All X is Y, no X is Y; 'all the ship's crew perished,' 'all the ship's crew survived.' In point of extent, this is the largest, the most sweeping and thorough negation, that can be advanced. The amount of knowledge required for such a denial, is at its maximum. It

is not often that, in dissenting from a Universal Proposition, we are able to substitute the opposite universal. We may doubt the truth of the affirmation 'all stars twinkle;' but we cannot carry our denial to the length of Universal Negation—'no stars twinkle.' Barely does any informed person, in advancing a universal proposition, go so far wrong, that the truth consists in the opposite universal.

There is the appearance of complete contrariety in the opposing views of the Immortality of the Soul. Christians say 'the souls of all men are immortal;' Buddhists and others say, 'no men's souls are immortal.' This, however, is one of the instances, where a universal is alike proved or disproved upon an individual case.

In small matters, total contrariety is frequent enough. The assertion may be made—'All the voters were bribed,' and may be met with the universal denial—'no voters were bribed;' which is felt to be the strongest denial that can be given.

Of this opposition, it is remarked, that both cannot be true, but both may be false. 'All men are wise' and 'no men are wise,' cannot be both true; the intention of the one is to declare the other to be false; between the two, there is a contradiction in terms. Yet it is possible that neither may be true, that *both* may be *false*. The truth may be neither the one, nor the other, but something betwixt the two sweeping universals; as, that some men are wise, and some not wise. Total contrariety, or complete negation, thus leaves room for a middle assertion.

It is farther pointed out in regard to this opposition, that the opposed propositions differ only in *quality*; the one affirms, and the other denies, of the same quantity, that is to say, the universal.

11. A Negation may consist in opposing a Universal Affirmative to a Particular Negative—A to O, or a Universal Negative to a Particular Affirmative—E to I. This is called the opposition of CONTRADICTORIES.

Instead of 'All men are wise,' 'no men are wise,' we may have the opposing couple, 'All men are wise,' 'some men are not wise; A and O. So, 'No voters were bribed' (E), is opposed by 'Some voters were bribed' (I). Such is contradictory opposition.

Of this opposition (as with contraries) both cannot be true; but farther, *both cannot be false*, or if the one be false the other must be true, and if the one be true, the other must be false.

There is not, as with contraries, an intermediate supposition ; there is no middle ground. Either 'all men are wise,' or 'some men are not wise ;' either 'no voters were bribed ;' or 'some voters were bribed.' The two opposites are so related that we must choose one or other. Hence to this kind of opposition belongs that principle first signalized by Aristotle, and ever since regarded as a primary Law of Thought—the LAW OF EXCLUDED MIDDLE.

It is farther noticed, that in contradictory opposition, there is change both in the *quality*, and in the *quantity* of the opposed assertions ; while one is affirmative and the other negative the one has universal, and the other particular quantity. This circumstance, however, instead of increasing, diminishes the contrariety. The change from universal to particular quantity abates the force of the opposition of quality.

The application of perhaps the strongest negative word in the language,—contradiction—to this kind of opposition calls for some comment. In common speech, the person that could, in reply to the charge—'All the voters were bribed,' maintain 'No voters were bribed,' would be held to have *contradicted* that charge in the most thorough-going way. While the declaration 'some voters were not bribed' would be regarded as a contradiction, the declaration—'no voters were bribed' would be held as a contradiction in a still higher degree. The word 'contrary' would be thought too feeble for universal denial.

It is apparent, that the logical contradictory, as now defined, denies much less than the logical contrary ; indeed, denies so little, that it excludes the possibility of a smaller denial ; it is the *minimum* of denial. For, whereas the affirmer boldly commits himself, for example, to the broad universal 'all men are wise,' the denier, timid and shrinking, ventures only upon an *exception* to the sweep of the rule ; he will not say, 'no men are wise,' which would be in common speech the flat contradiction, the thorough negation ; he merely says *some* men are not wise ; he denies so little, as to leave no room for any one to deny less. He takes ground so limited, so humble, as to exclude any more limited, more humble opponent. His 'some' commits him only to the fact of taking an exception. It may mean only *one* ; which of course would be an 'excluded middle,' for who that challenged the assertion 'all men are wise' could say less than 'one man is not wise ?' It is shaving the universal affirmative by the breadth of a hair that cannot be split.

The employment of the stronger term for the smaller opposition, is explicable thus. Aristotle, in dividing propositions

according to quantity—as universal and partial,—put great stress upon the difficulty in establishing, and the facility in subverting, a *universal*, whether affirmative or negative. The task of the affirmer is hard, he has to secure *every* individual instance; the task of the denier is easy, he has but to destroy *one*. If it were necessary, with a view to impugn a universal proposition, to establish an opposite universal, the difficulty of disproving an unsound generalization would be often insuperable. This, however, is not required. A single opposing fact is enough. A hole in a ship's bottom sinks her as surely as if she were torn plank from plank. It is this *sufficiency for disproof* that makes the importance of the limited contradictory affirmation. It can be more easily procured than the full contrary, and yet it is equally effective. It possesses the imposing circumstance of securing great ends by small means.

There are certain cases where the contrary and the contradictory are the same thing. The first is when the proposition is singular or individual: 'John is here'—'is not here,' 'The world was created in time,' 'The world is eternal,' There is no middle ground in such assertions as these.

Another case is where a generality stands or falls by an individual case, as in Laws of Causation. A single unambiguous observation (under what is called the Method of Difference) will prove Cause and Effect. If a new metal is discovered, and fused on one single occasion at 1100 deg. Fah., we may affirm generally that the same temperature will always fuse the metal. Here contrariety and contradiction are the same. The metal either is or is not fused at that temperature. The Uniformity of Nature prohibits the middle supposition, that some portions of the metal may be fused and some not.

These remarks serve to explain the use of the Law of Excluded Middle, by Sir W. Hamilton, in regard to certain questions, such as the Infinite Divisibility of Matter, Free-Will, the Eternity of the World. 'Matter is divisible,' 'matter is not divisible'—are contraries not contradictories; there may be a middle position—'some matter is divisible'—making them both false. But Hamilton must be understood to assume that Matter, either is a *singular* subject, or, is *homogeneous* to such an extent that what is true of one portion must be true of all, and consequently that the opposition above specified comes under contradictory opposition, which is governed by the Law of Excluded Middle. Accordingly, he maintains that of the opposite alternatives—matter is divisible, matter is indivisible; the will is free; the will is necessitated—one must be true and the other false.

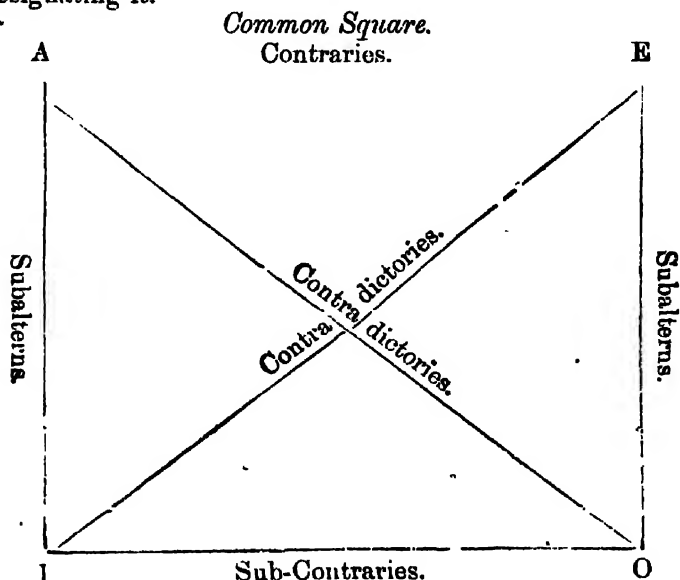
A farther logical convenience supposed to attach to the contradictory form is the substitution, for the denial of a universal,

of the equivalent, and corresponding affirmation. When A is denied, then, in that very act, O is affirmed. It being untrue that 'All men are wise,' it must be true that 'Some men are not wise.'

The Contrary and the Contradictory are the only important forms of opposition. It is usual to add another variety, that between a Particular Affirmative and a Particular Negative—I and O—'Some men are wise,' 'some men are not wise.' So imperfect is this opposition, that there need not be any contrariety between the two forms. They are compatible, and are often both true. All that can be said of them is, that they *cannot be both false*; if it is false that some men are wise, it cannot also be false that some men are not wise. But as the one predicate may relate to one set of men, and the other predicate to a different set, there is no real contrariety; frequently the two propositions together give the exact state of the case.

The name 'sub-contraries' has been given to these opposites. According to Hamilton, they were brought forward merely as completing the logical diagram, called the 'Square of Opposition.'

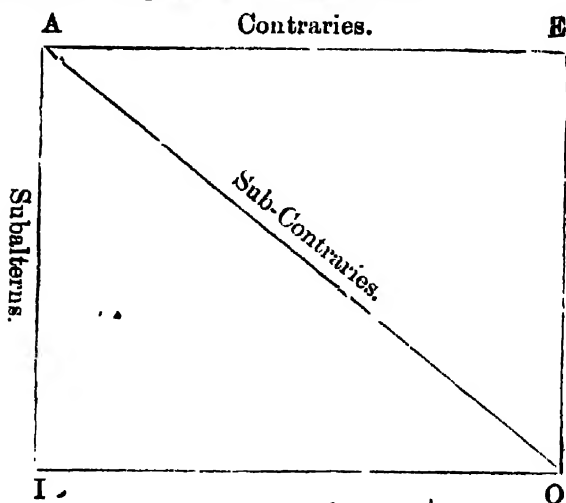
For the explanation of the diagram, it is farther to be remarked that the relation (which cannot be called opposition in the strict sense) between Universal and Particular—A and I, E and O, is called *subalternate*, or *subaltern*, the relationship of subordination. There is a sufficiently obvious propriety in so designating it.



Mr. De Morgan departs from this square on certain points. Regarding the words 'contrary' and 'contradictory' as the same in meaning, he drops 'contradictory,' and applies 'contrary' to the old meaning of contradictory, that is to the diagonal opposition, $A-O$, $E-I$. The opposition of the Universals, $A-E$, he proposes to style *sub-contrary*; and the opposition of the Particulars, $I-O$, which he retains, he calls *super-contrary*.

If we were to introduce any innovation of this nature, founded on the identity of contrary and contradictory in common speech, there would be a greater seeming propriety in the inverting of Mr. De Morgan's designations. The opposition of the Universals A and E —is full contrariety; the opposition of the Universal to the Particular of opposite quality (however effective as a logical instrument) is still but *partial* contrariety, or subaltern contrariety, and would better suit the name 'sub-contrary.' $A-O$, $E-I$. The opposition of the particulars I and O does not, so far as can be seen, need any descriptive name. If it did, 'super-contrary' might be taken.

The supposed square would stand thus :—

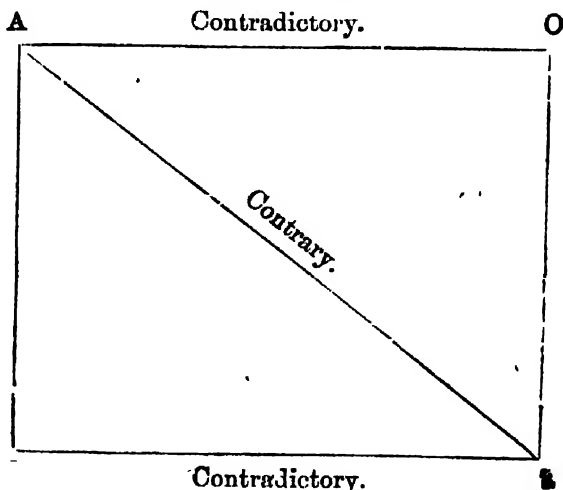


This form is the following out of the view already taken of the imperfect negation of the so-called contradictories. It is also so far in harmony with the scheme of the diagram (borrowed from the Parallelogram of Forces), a superficial harmony founded on a deeper propriety. Thus, A E , being one

side of the square, and the line of the subalterns, A I, being the side adjoining; the composition of these two, into the diagonals, A—O, or E—I, yields *subaltern contraries*, contracted into *sub-contraries*. This is not a mere accidental coincidence of language; it is the expression of the fact that subaltern or subordinate contrariety, is a subordinated, narrowed, or partial form of contrariety; a whole is opposed, not by a whole, but by a part; a universal met, not by a universal, but by a particular; giving a diagonal or oblique contrariety, instead of a full or total contrariety.

This is different from the common square, as well as from the two others given above. Aristotle uses the diagonal for the full contrary opposition of the two universals A and E. The contradictories, or sub-contraries, A-O, E-I, are the sides (between right and left). There is no opposition indicated between A and I, E and O; and the second diagonal is left blank, inasmuch as I and O, are not proper contraries. This square has the diagrammatic property of representing the strongest contrariety by the longest line, the line also that bisects the figure; from which arrangement arose the emphatic phrase *diametrical* opposition, to signify the thorough opposition of the universals.

Aristotle's Square.



Modal Propositions.

12. Since, in common speech, Propositions often occur in a qualified or modified form, a class was constituted by Aristotle for such cases, under the name of MODAL Propositions; the unqualified forms being called the *Pure* forms.

If we were to say that, in Geometry, 'the conclusion *necessarily* follows from the premises,' the affirmation would be called Modal; it lays down a truth and farther designates it as a *necessary* truth. The contrast of necessary is *contingent*, which is also a modal; the propositions of physical science are looked upon as not necessary, but contingent; the facts might have been arranged otherwise. So that besides affirming that oxygen combines with hydrogen, we might call it a 'contingent' doctrine or statement. Other generic forms of modality, included by Aristotle, are the *possible*, and *impossible*; both which may qualify propositions. He reduces these four forms to two,—*necessary* and *contingent*. He was supposed also to have taken in *true* and *false* among the kinds of modality. Although this is doubted by some, there would be no reason why they should not be included. So, *probability* and *improbability* might be likewise admitted. Subsequent logicians extended the species of modality to qualifying adjectives or adverbs, as 'the *white* man runs,' 'he runs *quickly*.' Again, the qualification of *time* is an important fact entering into many propositions; he *ran yesterday*; he *continues* running.

That such propositions are frequently to be found is obvious. By Hamilton and the stricter of the formal logicians they are excluded from Logic. They clearly do not belong to the narrow Formal or Syllogistic Logic. They have reference to the *matter* and not the form of predication. They are included in the more comprehensive Logic sketched in this work; and we can easily assign their proper position in the enlarged scheme. Propositions qualified as *Necessary*, first give an affirmation, and secondly, declare that such affirmation belongs to the class of necessary truths, whatever these may be; whether this be true or false depends on a comparison of the marks of the class 'necessary truths,' or the connotation of the word 'necessary,' with the affirmation in question. The case falls under Deductive Evidence, not formal, but material, like the interpretation of Law. The same remarks apply to Contingent, Possible, and Impossible propositions. With regard to *Proba-*

bility, as a modal, a reference would be made to the branch of Induction treating of Probable evidence.

Propositions qualified by present, past, or future time, or in any of the tenses of the verb besides the present viewed as the universal tense, may be treated as compound propositions; asserting first a fact, and then the time of its happening. Another view of these, suggested by Mr. Mill, is to associate the tense with the copula.

In the Appendix (EXPLANATION OF TERMS, *Modals*) will be given the usual statement of the Opposition of Propositions, as applied to Necessary, Impossible, and Contingent matter. It is withheld from the Text, as being an irrelevant and useless complication.

IMPORT OR MEANING OF PROPOSITIONS.

13. For laying out the divisions of the Inductive Logic, it is requisite to classify propositions according to their IMPORT or Meaning.

Although the special meanings of propositions are as various as human knowledge, there are certain highly generalized meanings, pointing to difference of Logical Method.

14. To the question, what is, in matter or *substance* (as contrasted with *form*), the meaning of a Proposition, Hobbes answered that, in a proposition, the predicate is *a name for the same thing as the subject is a name for*.

Thus, 'Aristides is just' is a true proposition if 'just' be the name of Aristides. 'Men are gods' is false, because 'god' is not a name for man.

This is true, but not the whole truth. The theory is correct so far as it goes, but it does not reach to the final import of predication. Hobbes did not advert to the real meaning, which is found in the *connotation of class names*. When we say, 'Aristides is just,' the preliminary question arises, how came the name 'just' to be applied to Aristides? When the word was first determined on, people knew nothing of Aristides. What they knew was the agreement of a certain number of persons in a peculiar feature of conduct; to that agreement was given the name 'just.' Any one in after times found to have the agreeing feature, succeeded to the name; and the meaning of the proposition as regards Aristides is that he *resembled* a number of persons that went before him, in a certain point where they resembled one another; and on

account of which, they were named 'just.' In one view, therefore, the proposition in question is an affirmation of *likeness*; but that fact must enter into every proposition asserting participation in a community of attributes. More characteristic of the case is the feature of *co-existence*; the co-existence of the man Aristides with the quality named 'just.' Two things are mentioned; and these two things are united in predication, by declaring their co-existence in one subject. Whether this is a typical or representative instance, will be seen, after a fuller examination of particulars.

15. A second theory, sharing in the same defect as the foregoing, is that Predication consists in *referring something to a class*,—placing an individual under a class, or one class under another class.

When we say 'the planets are round,' on this hypothesis the meaning would be, 'the class planet falls under, or is enrolled in, the class round;' 'Neptune is a planet,' Neptune is in the register of bodies named planets. Or, negatively, 'men are not gods,' men are not to be found in the list of the gods. This is both inadequate and incorrect. It confounds the connotation of a name with its denotation; the class *attribute*, which is elastic and indefinite, with the class as supposed to be an aggregate of definite individuals. The meaning of a general name is as extensive as the things that possess the attribute; although a certain number of known individuals may be recognised as a group; or class, corresponding to the name, the class must ever remain open to new individuals. We have a general name 'sea,' which is also a class name, in the narrow sense. The individual seas of the globe are enumerated in geography; but these are not exclusive. We could not refuse the name 'sea' to a newly discovered individual, because it is not in the old list; if it possessed the common features, we should give it the name at once, and write it down in the list afterwards. Most general names have no lists or registers of individuals; we have no exhaustive tables of round things, of stars, of coal strata, of whales, or of human beings. We have merely points of agreement, defining marks; in other words, a meaning or connotation to each term; the correspondence with this rules the application of the word, or the truth or falsehood of the proposition asserting that any individual is round, is a star, and so on.

In forming a class, we do not, as in forming a society, enroll certain definite individuals, and decide each one's

pretensions by referring to the roll. We indicate an attribute or attributes, and test the individual by the presence or the absence of the attributes.

16. There are two ways of arriving at the highest generalities of Predication. One is a sufficiently wide examination of actual propositions in the detail. The other is to refer to the classification of 'Nameable Things.' The two modes should confirm each other.

By an examination of propositions in detail, we should soon find many of the kind already noted as affirming Co-existence; the co-existence of two things, or facts, or two properties. 'Man is mortal,' is the co-existence of humanity and mortality. 'The fall of the barometer is a sign of rain,' is the concurrence of the two facts, the fall of the barometer and rain.

We might then turn from co-existence, to its contrasting property, 'Succession,' and enquire whether any propositions are made up of two or more things affirmed to happen in succession. We should find many such. 'The wind raises the sea,' 'the sun is the cause of vegetation,' 'Cæsar subverted the Roman Republic,' might all be interpreted as affirmations of succession. Speaking generally, wherever there is production, causation, or change, there must be succession; one state of things is followed by another state of things. In cause and effect, which is a very wide department of human enquiry, there is understood to be succession; something called a cause is followed by some other thing, called an effect.

We have seen, farther, that in predication, there is involved the declaration of Likeness and Unlikeness. This contrast, however, is a universal fact inseparable from predication; the very basis of cognition is laid in Difference and in Agreement. But there are certain cases where the specializing point of a proposition lies in likeness or unlikeness; as in propositions of Number. 'Twice two is four' is an affirmation of Equality; the test of its truth would be a test suited to ascertain equality or inequality. It could not be brought under either co-existence or succession in an easy or natural way; it falls readily and fitly under agreement or disagreement in respect of Quantity.

17. A reference to the classification of Nameable Things shows the wide compass of these three affirmations—Co-existence, Succession, and Equality or Inequality.

Under Nameable Things (APPENDIX C), we find attributes

special to the Object, attributes special to the Subject, and attributes common to both. The attributes common to both are Quantity, Co-existence, and Succession. We might, on the strength of this enumeration, give, as universal forms of Predication; attributes of the Object, and attributes of the Subject, declared as agreeing or disagreeing in *Quantity*, as *Co-existing*, or as *Successive*.

18. I. Propositions of QUANTITY include the whole of the Mathematical sciences, and all the applications of number to quantity in every science and art. The predication is *equality* or *inequality*.

Thus, in Arithmetic, the addition and subtraction of numbers, the multiplication table, and the rule of three,—which are the fundamental processes—are affirmations of agreement or disagreement in quantity. Three and four is seven; five from nine leaves four; six times eight is forty-eight; as two is to ten, so is six to thirty,—are affirmations of equality or agreement in numerical quantity.

The propositions of geometry may all be resolved in like manner. The angle in a semi-circle is equal to a right angle. A sphere is equal in bulk to two thirds of the circumscribed cylinder. Two sides of a triangle taken together are greater than the third (Inequality).

In Algebra, we need allude only to the extensive process of manipulating by *Equations*.

In every art and in every emergency of life, occasion arises for measuring quantity, that is for declaring equality and inequality, greater or less. Even when the quantity does not admit of numerical statement, as in shades of feeling and of human character, we still express and compare quantity; we call one man more energetic, more far-seeing than another.

19. It is the characteristic of the Sciences of Quantity to be purely Deductive Sciences. They have Inductive foundations like all the rest, but the chief labour attending them consists in purely deductive operations.

This determines the Logical Method and the Logical Department of Mathematics. All that is peculiar in the science belongs to the branch of Logic named DEDUCTION.

20. II. Propositions of CO-EXISTENCE are of two kinds. In the one kind, account is taken of Place; they may be described as propositions of *Order in Place*. They refer purely to the Object, or Extended World.

The Object, or Extended Universe is a vast array of things distributed in space; they are said to have *place*, or a mutual relationship as to extension. Thus, the stars are arranged in the celestial vault at definite distances. Geography is a body of propositions of order in place; an ocean, a mountain chain, a river—are described geographically as having local situation with reference to other things; to these are applied the more purely mathematical or quantitative propositions of magnitude.

Some propositions of Place affirm nothing beyond containing and contained; they declare one thing to be either in or out of another thing;—John is in the room; the constellation Orion is in the northern hemisphere; St. Helena is in the South Atlantic; The British Museum contains the Portland vase. These may be called the more vague and indeterminate propositions of quantity. The degree of precision, in this case, depends upon the relative magnitudes of the container and of the contained. A thing affirmed to be in a house is better defined than a thing in a town, and not so well as a thing in a drawer.

Another mode of giving order in place is to affirm close proximity. One thing outside another, but in contact with it, has a definite position, expressed by such forms as 'by,' 'by the side of,' 'close to,' 'above,' 'beneath.' If there be an interval, a measured distance must be assigned.

The more precise propositions of Order in Place are those that declare mutual position by numerical statements of distance or extension; to which form every fact of order in place might be reduced, if we had sufficient knowledge, and if we thought it necessary or desirable. Thus, the mutual position of the stars, in the sphere of the sky, is stated in terms of angular measurement; the position of places in the earth is given by latitude and longitude, and also, if need be in linear distances. The determination and the expression of this relationship, therefore, may be wholly referred to Arithmetic and Geometry. The precise statement of relative position is the peculiar province of Analytic or Co-ordinate Geometry.

The description of all objects of the external world having parts, or a defined structure, demands propositions of Order in Place, according to some one of the foregoing methods; as buildings, machinery, plants, animals, aggregates and collections of objects.

21. The second form of Co-existence may be designated *Co-inherence of Attributes*.

This is a distinct variety of Propositions of Co-existence. Instead of an arrangement in place, with numerical intervals, we have the concurrence of two or more attributes or powers. in the same part or locality. A mass of gold contains, in every atom, the concurring attributes that mark the substance—weight, hardness, colour, lustre, incorrosibility, &c. An animal, besides having parts situated in place, has co-inhering functions in the same parts, exerted by the very same masses and molecules of its substance. Every blood corpuscle has a plurality of relations, indivisible and inseparable.

The Mind, which affords no propositions of Order in Place, has co-inhering functions. We affirm mind to contain Feeling, Will, and Thought, not in local separation, but in commingling exercise. Every pleasurable feeling has its power of acting on the will and of impressing the memory; all the attributes are joined in the unity of the mental being.

A wide range of Scientific knowledge is comprised under the present head. The concurring properties of minerals, of plants, and of the bodily and the mental structure of animals, are united in affirmations of co-inherence. The investigation of these concurrences, whether special or general, is a branch of scientific method, or of Logic, coming under INDUCTION, although not the largest portion of the Inductive department.

22. III. Under SUCCESSION, there are also two kinds of Propositions. By the first is predicated *Order in Time*.

This is Parallel to Order in Place, under Co-existence. Many propositions consist in assigning the order and sequence of events, without intimating any closer relationship. The world being constituted on the principle of change, there is a serial order in its phenomena, which may be given in narration. Spring is preceded by winter, and succeeded by summer; infancy is followed by youth. The treaty of 1815 followed Waterloo.

The position of events may be defined by their *close* succession. First the seed, then the ear, then the full corn in the ear. Henry VIII, succeeded Henry VII, and preceded Edward VI. A serial order being given, the position in the order is fixed either by contiguous events, or by a numerical position, as the sixth Earl.

Here, too, as in order in place, the precise method consists in the use of numbers. The flow of time being divided into years, months, days, hours, &c., the position of any occurrence is given by numbers and by fractions of numbers. This is merely another application of Arithmetic. In the complica-

tions of Astronomy, the element of time may require difficult algebraical formulæ. There is, however, no new and distinct department of scientific enquiry involved in propositions of mere sequence in time, however accurately they may be investigated and recorded.

23. The second mode of Succession, is that denominated *Cause and Effect*. The largest part of Induction is occupied with this department.

Cause and Effect appears under the guise of Succession, but contains something beyond the sequences above considered. There is supposed to be a certain bond or *nexus*, a determining power or agency, whereby the one gives birth to the other. Propositions of Cause and Effect are such as these:—the explosion of gunpowder propels a cannon ball; the combustion of coal converts water into steam; light is an agent of decomposition; anxiety wears the constitution; a good harvest makes prices fall; Demosthenes incited the Athenians against Philip.

The Logic of Induction is occupied first with propositions of CO-INHERING ATTRIBUTES; secondly, and mainly, with propositions of CAUSATION. Although the foundations of the science of *Quantity* are also inductive, yet so limited and simple is the induction, that it may be sufficiently noticed in the account given of this department under Deduction and the Deductive Sciences.

The foregoing is a modification of Mr. Mill's scheme of the Import of Propositions in the final analysis, conceived with the view of ascertaining the divisions of Logic.

Mr. Mill enumerates five ultimate predicates, or classes of predications—EXISTENCE, CO-EXISTENCE (including Order in Place), SUCCESSION, CAUSATION, RESEMBLANCE.

Apart from Existence, these are substantially the classes here adopted. CO-EXISTENCE, as explained by Mr. Mill, comprises Order in Place, and also the Properties of Kinds (Book III. Chap. XXII), which are given above under 'co-inhering attributes.' By SUCCESSION, is meant the looser successions included under Order in Time. The successions of Cause and Effect are given in a distinct and co-ordinate predicate—CAUSATION. Under RESEMBLANCE, Mr. Mill indicates propositions expressing the identity of the things discovered to be identical, as, for example, in classification; but this underlies all propositions where there is generality, and does not mark off a scientific department. In the end, however, he gives as the

special science of Resemblance, propositions of Quantity, or Mathematics.

With regard to the predicate *EXISTENCE*, occurring in certain proposition, we may remark that no science, or department, of logical method, springs out of it. Indeed, all such propositions are more or less abbreviated, or elliptical; when fully expressed they fall under either co-existence or succession. When we say there *exists* a conspiracy for a particular purpose, we mean that, at the present time, a body of men have formed themselves into a society for a particular object; which is a complex affirmation resolvable into propositions of co-existence and of succession (as causation). The assertion that the *dodo* does not exist, points to the fact that this animal once known in a certain place, has disappeared or become extinct; is no longer associated with the locality: all which may be better stated without the use of the verb 'exist.' There is a debated question—Does an Ether exist? but the correcter form would be this—'Are heat and light and other radiant influences propagated by an etherial medium diffused in space;' which is a proposition of causation. In like manner the question of the Existence of a Deity cannot be discussed in that form. It is properly a question as to the First Cause of the Universe, and as to the continued exertion of that Cause in providential superintendence.

EQUIVALENT PROPOSITIONAL FORMS—IMMEDIATE, OR APPARENT INFERENCE.

24. Great importance often attaches to the equivalent modes of expressing the same fact, assertion or proposition. The transforming of one expression to another is so far an aid to reasoning as to be sometimes termed 'Inference.'

The enumeration of Equivalent Forms is as follows:—

- I. Universal and Particulars.
- II. Greater and less in Connotation.
- III. Obversion.
- IV. Conversion.
- V. Hypothetical Inference.
- VI. Synonymous Propositions.

The first to the fifth, inclusive, are each conducted on a definite plan, admitting of precise rules. They are, therefore, the properly logical modes. The sixth,—Synonymous expression—is indefinite and various; so that, although deserving of notice, it is not reducible to rule.

It will appear, in the course of the exposition, that in none of these cases is there Inference properly so called, that is to say, the transition from a fact to some different fact; there is merely the transition from one wording to another wording of the same fact. Hence, the designations '*Immediate Inference*,' and '*Apparent Inference*,' to distinguish the process from Mediate or Real Inference.

Universal and Particulars—Greater and Less in Denotation.

25. A Universal Proposition and its constituent particulars being the same, there is no real inference, but a repetition, in saying All A is B, therefore Some A is B; all men suffer, therefore some men suffer.

A Universal Proposition is the summed up equivalent of many particular propositions, and has no force beyond, or apart from the particulars. Hence, when we state a particular case, we do but resolve the universal into its elements, and take these individually as they were before the universal was formed. 'All the houses of the street are newly built' is a mere summary or abbreviation of the separate enumeration—No. 1 is new, No. 2 is new, and so on. To say 'all the houses are new,' therefore 'No. 6 is new,' is not to make an advance in knowledge, but to fall back upon one of the constituents of the general proposition. The law of Consistency requires that whoever asserts a fact universally must be prepared to abide by it in each particular instance. A shopman advertises a number of articles at a shilling each; the buyer, taking him at his word, chooses some one article, and puts down a shilling.

Greater and Less in Connotation.

26. In regard to the Connotation or Comprehension of a term, it is no inference to affirm the less after assuming the greater.

When we say 'John is a man,' we say that he has each and all of the properties connoted by, or comprehended under 'man.' It is no new affirmation, therefore, but merely unfolding in the detail what is already summed up in the aggregate, to say John is a living creature, an animal, a compound of body and mind. Whoever is not prepared to admit these affirmations, should not declare John to be a man.

In maintaining that 'quadrupeds are endowed with mind,' we hold that they possess Feeling, Will, and Thought. It

is, therefore, not a real inference but a mere iteration, to add 'quadrupeds feel,' 'quadrupeds will.'

When we affirm that a certain substance is arsenic, we affirm of it all the known properties of arsenic. It is an equivalent or identical proposition to say, 'the substance is poisonous.'

These affirmations of the properties of things in the detail have already come under our notice, as verbal, essential, or identical propositions.

We must consider ourselves at liberty to join or disjoin the attributes of a thing, without real inference. We may say either 'Socrates was wise, virtuous, and a martyr,' or 'Socrates was wise,' 'Socrates was virtuous,' 'Socrates was a martyr.' Given an aggregate or compound proposition, we may reduce it to its elements; given a number of elementary propositions, we may compound them into one. The operation lies more in the grammar than in the sense.

'Socrates was virtuous,' 'there was one man virtuous,'—may be held to be a purely equivalent form. If we enquire into the meaning of the word Socrates, we find 'among other things' that it means 'a man,' 'one man,' and to say 'one man was virtuous' is no new meaning, but a part of the original meaning. So, after saying, 'Socrates was virtuous' and 'Socrates was poor,' there is no inference in saying 'one man was virtuous and poor,' or 'one poor man was virtuous.' This example has some importance in the theory of the Syllogism.

Under the designation—Immediate Inference by *Added Determinants*, the following case is given (Thomson's *Laws of Thought*);—'A negro is a fellow-creature; therefore a negro in suffering is a fellow-creature in suffering.' This seems self-evident, but it is somewhat different from the other cases. It resembles the following mathematical inference: $A = B$, whence $A + C = B + C$; which is not an immediate judgment, but deductively inferred from the axiom—'The sums of equals are equal.'

Even allowing the axiom of addition of equals for such a case, we must be cautious in applying it without regard to the matter, seeing that the same addition may not have the same effect upon both sides. 'Beauty is pleasure; hence beauty in excess is pleasure in excess,' is not a safe inference; the qualification does not operate precisely alike upon both subjects.

Obversion.

27. In affirming one thing, we must be prepared to deny the opposite: 'the road is level,' 'it is not inclined,' are

not two facts, but the same fact from *its other side*. This process is named **OBVERSION**.

On the principle of Relativity, every statement has two sides, as a part of its nature : there is always something to be denied when any one thing is affirmed. Whoever is 'wise' is 'not foolish ;' we must grant both propositions or neither. In this we make no march, no addition to our knowledge ; the utmost that we do is to give completeness to the statement, there being usually an ellipsis or omission of the co-related fact. 'This end of the magnet is not the north end ; therefore it is the south end,' is no inference ; if it is not north, it is, by necessary implication, south. 'I don't like a curving road, because I like a straight one,' is a childish reason, being no reason at all, but the same fact in obverse.

To each of the four Propositional Forms, A, I, E, O, there is an obverse form :—

Thus, in A,

Every X is Y ; every man is mortal,

We first *obvert the predicate*,

Every X is not Y ; every man is immortal,

And next *prefix the sign of negation*.

No X is not Y ; no man is immortal.

So, all inert matter gravitates, no inert matter (not-gravitates) fails to gravitate. All gold is precious, no gold is (not-precious) worthless. All virtue is profitable, no virtue is (not-profitable) useless, devoid of utility. Freedom of Trade tends to peace ; freedom of Trade averts war. All knowledge is useful ; no knowledge is useless.

To obvert I,

Some X is Y ; some men are wise,

Obvert the predicate, and prefix the sign of negation

Some X is not not-Y ; some men are not (not-wise) foolish.

Some stones are precious ; some stones are not (not-precious) worthless. Some virtues are burdensome ; some virtues are not (not-burdensome) easy.

For E,

No X is Y, no men are gods.

The obverse is,

All X is not-Y ; all men are no-gods (excluded from the gods).

No crows are white ; all crows are excluded from white things, are of some other colour than white ; or, if the universe of the predicate 'white,' be not colours, but white and black, 'all crows are black.'

The rule here is the opposite of the rule for A: *obvert the predicate, and remove the negative sign.*

The obverse of O,

Some X is not Y; some men are not wise

Some X is not-Y; some men are (not-wise) foolish.

Some of the crew were not saved; some were (not saved) lost.

The rule still is *obvert the predicate, and remove the negative sign*, which is to change the quality of the proposition.

The Universal affirmative with *universal quantity* in the predicate,—

All X is all Y; all inert things are all gravitating things, is obverted to the same form as the obverse of A.

No X is not-Y; no inert things are found among things that do not gravitate.

All equilateral triangles are all equiangular triangles; no equilateral triangles are to be found among triangles with unequal angles. All double-refracting bodies are all bodies that polarize light; no double-refracting bodies are to be found among bodies that do not polarize light.

The Particular Affirmative with a universal predicate, Y has the same obverse as I. Some X is all Y: some mortals are all men. Some X is no not-Y; some X is not to be found among not-Ys. Some mortals are not to be found among objects that are not men. There is a class or group of mortals that you will not discover among the brutes (Universe Animals), among the plants (Universe organized bodies).

Material Obversion.

28. There are Obverse Inferences justified only on an examination of the matter of the proposition.

From 'warmth is agreeable' we can affirm, by formal obversion, 'warmth is not disagreeable, and not indifferent.' We cannot affirm, without an examination of the subject-matter, 'cold is disagreeable.'

There is a mode of inference, included by some logicians among Immediate Inferences, whereby we might say, 'the absence of warmth is the absence of an agreeable thing.' This granted, we are still a good way from 'cold is disagreeable.' We must be able to say farther—'the absence of warmth is the same as cold, and the absence of the agreeable is the same as the disagreeable.' But we are not entitled to say this, except on a reference to the fact; and such a reference teaches us that the absence of warmth may not be the same as cold.

and the absence of the agreeable not the same as the disagreeable; there is a possible neutral state in both cases. But the same experience teaches us that in an actual state of pleasurable warmth, the sudden change to cold is also a change to the disagreeable. Whenever an agent is giving us pleasure in act, the abrupt withdrawal of that agent is a positive cause of pain. On the faith of this induction, we can obvert materially a large number of propositions regarding pleasure and pain, good and evil. If the sight of happy beings gives pleasure, we may infer, not by formal implication, but by material or real inference, that the sight of unhappy beings gives pain. The inference is a consequence of the laws of our sensibility. While the sight of happy beings is giving us actual pleasure, any sudden withdrawal or disturbance of that sight is a painful shock or revulsion. What is more, the organization formed to take pleasure in happy beings, is by that very circumstance formed to take pain at the sight of the unhappy. So we cannot take pleasure in opposing facts--praise and blame; we cannot become indifferent to the one without becoming indifferent to the other.

From 'War is productive of evil,' we cannot say by formal obversion, 'Peace is productive of good.' As before, 'the cessation of war is the cessation of an evil,' and is therefore good, in accordance with the law of our sensibility that the remission of a felt pain is a pleasure.

It is a true inference, but not a formal implication, that if an upright minister gives public confidence, a shuffling minister causes mistrust. Provided the public confidence is owing to the minister's uprightness, the replacing of that quality by its material opposite must produce the opposite of confidence.

The remark is sometimes made, 'government has great power for evil, and but little power for good.' Rigidly examined, this is a contradiction. He that is able to do us a great harm is able to refrain from that harm, and to make all the difference in our lot between our present tolerable condition and a condition of intolerable misery. The saying is true to this extent, that government interference, exerted for bad, could cause more misery than the same interference, exerted for good, could cause happiness.

'Cold kills animals,' does not necessitate 'heat keeps them alive.' By a material inference from the law of causation, we are entitled to say, keep away the cold that kills, and, so far as that agency is concerned, the animals will live. This is not formal implication; it is a certainty grounded on causation.

'Force compresses bodies,' does not justify 'the withholding of force expands them.' We can say only, 'the absence of force leaves bodies in their uncompressed state.' This, in like manner, is a material inference from causation.

If 'knowledge is good,' we must concede the obverse, 'ignorance is bad,' but not by formal implication. Whatever amount of good, knowledge, as knowledge, is capable of doing, must be lost according as knowledge is withheld.

Aristotle says, 'the beneficent man loves those he has done good to.' There is a familiar saying that may be given as a material obverse, 'we hate those we have injured.' By the laws of our sensibility, the two facts are mutually involved; although there are limitations that we learn by an induction from the facts.

Conversion.

29 The Logical doctrine of the CONVERSION of Propositions is a case of equivalence. In Conversion, the Subject and the Predicate of a Proposition exchange places.

The Proposition X is Y converted, becomes Y is X; X is not Y, Y is not X; men are mortals, mortals are men.

The simple reversal of subject and predicate does not always give an equivalent form: 'all men are mortals' is not the same as 'all mortals are men.' This arises from the circumstance—taught us by our knowledge of things, and not discoverable by the examination of forms—that there are other mortals besides men. In all such propositions, therefore, a qualification must go along with the reversal of the terms.

(1) In the forms E and I, the reversal of the order of the terms needs no qualification. Accordingly, this is termed unqualified, or *Simple Conversion*. 'No X is Y,' is commutable into 'no Y is X,' without alteration of meaning. If 'no men are gods,' 'no gods are men;' the proposition declares mutual exclusion or incompatibility, and we are at liberty to signify the exclusion from either side; X excludes Y, and Y equally excludes X. No crows are red; no red objects are crows. No chemical combinations take place in fluctuating proportions; no combinations in fluctuating proportions are chemical.

In I, 'Some X is Y,' 'some minerals are crystals,' we can say, by simple reversal, 'Some Y is X, some crystals are minerals. Some water is pure, some pure material is water. It is as when two areas cover one another partially; the partial coincidence is expressed from either side without change of signification.'

In a simple conversion of this nature, 'some' has a different value in the two propositions, unless the predicate and the subject are co-extensive. Thus, in the couple,—'Some men are dark-haired,' 'some dark-haired beings are men,'—'Some men,' as compared with 'all men,' is a larger fraction than 'some dark-haired beings,' as compared with 'all dark-haired beings.'

(2) In converting A, the universal affirmative, 'All X is Y,' 'all fires give heat,' we have to qualify or limit the subject, *Some Y is X*, *some* sources of heat are fires. There may be other Ys besides the Xs, and other sources of heat besides fires; so that we must leave the possibility open, which would not be done in simple conversion—(*all Y is X*, *all* sources of heat are fires). To this *qualifying* conversion, logicians apply the designations *Limitation*, and *per accidens*. The Greek original of Aristotle was more descriptive, *κατὰ μέρος*, 'partitive' conversion.

One of the recommendations of the thorough-going quantification scheme of Hamilton, is that it anticipates this necessity of qualifying the new subject. The proposition being expressed, in the first instance, as All X is some Y, or all X is all Y, as the case may be, the converse is Some Y is all X, or all Y is all X. 'All men are some frail things;' some frail things are all men.

By far the most fertile source of purely syllogistic fallacies is the tendency of the mind to convert universal affirmatives without limitation. The usual form of the language, All X is Y, unless we are specially put on our guard, is apt to be interpreted, as if X and Y were co-extensive; in other words, we are disposed to regard it as justifying the simple conversion, all Y is X. The errors of syllogism to be afterwards pointed out, under such names as Undistributed Middle, and Illicit Process, mostly grow out of this subtle error of conversion. When it is said, 'All powerful minds have large brains,' the hearer readily slips into the unlimited converse, 'All large brains indicate powerful minds.' This fallacy of conversion is of frequent occurrence; and there is no more useful application of Logical forms than to warn against it. The best warning, however, consists in multiplying examples to show that, in universal affirmative propositions, the subject and the predicate are very rarely of equal extent; and that, when they are equal, it is usual to make known the fact by some form of language.

A few instances are subjoined. 'Ill doers are ill dreaders,' does not suppose that 'Ill dreaders are ill doers'; there may be many causes of dreading evil, besides having done evil.

'All protestants exercise the right of private judgment;' so do *other* persons *besides*; hence we cannot say that whoever exercises private judgment is a protestant.

'All beautiful things are agreeable'; beautiful things, however, do not exhaust all that is agreeable; there are more agreeable things than there are beautiful things.

'All virtue conduces to the good of mankind;' it does not follow that whatever conduces to the good of mankind is virtuous. 'The good of mankind' is a much wider meaning than virtue.

'All the pleasures of the imagination,' says Addison, 'arise from the great, the uncommon, and the beautiful.' He must be supposed to mean that the sources of these pleasures are found among things that are great, among things that are uncommon, and among things that are beautiful. But the classes 'great' and 'uncommon' must contain many objects besides those yielding imaginative pleasure. If this is not the case with the 'beautiful,' it is because 'beauty' and 'imaginative pleasure' are almost synonymous.

When Sir G. C. Lewis remarks that 'Historical evidence requires *contemporary registration*,' he does not mean that contemporary registration will of itself make historical evidence. This is one condition, but there are other conditions besides.

The universal affirmative, when stated in Comprehension, or Connotation,—'the property A is accompanied by the property B,' 'the attributes of man are accompanied by attributes mortal,' is the form least favourable to suggest a limited or qualified conversion. We are still more disposed than with the form of Extension, to convert simply;—'the attribute mortal is accompanied by the attributes of men.' Hence, for all the purposes of the Syllogism, the proposition in Extension is alone useful; the fact being borne in mind, however, that the Extension is *determined* by the Connotation.

(3) In converting O, the Particular Negative, (Some X is not Y, Some men are not Englishmen) a complex operation is necessary. Simple conversion—Some Y is not X, Some Englishmen are not men—does not apply. Two steps have to be gone through, first, *obversion*, and secondly, simple conversion.

Thus, by obversion,

Some X is not-Y (something that is not Y),

Some men are not-Englishmen (out of the class Englishmen).

These obverted forms are Particular Affirmatives, and are therefore converted simply:—

Some not-Y (something not Y) is X.

Some beings that are not Englishmen, are men.

'Some men are not wise.' By obversion,

Some men are not-wise (foolish).

By simple conversion,

Some foolish beings are men.

The names given to this compound process are conversion by *Negation*, or *Contraposition*. It might also be called *Obverted Conversion*.

A similar operation may be performed upon A, the Universal Affirmative, so as to yield an equivalent negative form with transposed terms. The reduction of the syllogistic mood named *Buroko*, requires this operation.

Thus,

All X is Y,

gives, by Obversion,

No X is not-Y.

which, by simple conversion (of E), is

No not-Y is X.

Or,

All men are mortal

No men are immortal

No immortals are men.

In the same way, 'All the righteous are happy,' is converted into 'No unhappy persons are righteous.'

Hypothetical Inference.

30. Hypothetical Propositions are of two kinds—Conditional and Disjunctive. They have been treated as the basis of a distinct form of Syllogism, called the Hypothetical Syllogism.

If the education of children is neglected, they will grow up ignorant,' is regarded as the major premise of a syllogism; and by adding, as minor, 'now certain children have been neglected,' we are entitled to the conclusion, 'they will grow up ignorant.' This has been called a Hypothetical Syllogism (Conditional). By a Disjunctive Proposition (A is either B or C), coupled with a proposition giving one alternative (A is not B), we seem to infer the other alternative (A is C); which would be a Disjunctive Syllogism.

In his Lectures on Logic, Sir W. Hamilton, following the usual practice, takes up hypothetical reasoning after Syllogism; but in the notes at the end, published after his death, he prefers to treat it as a case of Immediate Inference. Mr. Mansel, also, argues that

hypothetical reasoning, so far as it is purely logical, is purely categorical. The obvious differences between the syllogism and hypothetical reasoning are (1) the absence of a middle term; in the hypothetical syllogism all the terms are introduced in the so-called major; (2) the minor and the conclusion indifferently change places, and each of them is merely one of the two members constituting the major; (3) the major (so-called) consists of two propositions, the categorical major of two terms.

The Conditional form applies in the first instance to cause and effect. If the cause is present the effect is, and if the effect is absent the cause is absent. But the same form holds good when one thing is the sign of another, or is constantly associated with that other.

Boole and De Morgan are of opinion that the hypothetical inference is not different from immediate inference. Boole observes in his 'Laws of Thought' (p. 241) that the hypothetical syllogism is no syllogism at all, as it need contain no more than two terms. De Morgan says—'The law of thought connecting hypothesis with necessary consequence is of a character which may claim to stand before syllogism, and to be employed in it, rather than the converse.' (Syllabus, p. 66).

31. In the CONDITIONAL Proposition—If A is B, C is D, the equivalent is—A being assumed to be B, it follows that C is D.

There is no inference in this case. Accepting 'A is B,' we accept 'C is D;' this is another expression for the same fact. 'If the weather continues fine, we shall go to the country,' is transformable into the equivalent form 'The weather continues fine, and so we shall go to the country.' Any person affirming the one, does not, in affirming the other, declare a new fact, but the same fact. No new matter is introduced into the assertion; it is a pure instance of the Law of Consistency. 'When a buyer offers a seller a certain price for an article, and the seller says,—Here, then, is the article—the buyer is only consistent with himself in paying the price. Yet this is all that is done in a supposed conditional inference.

A second form of so-called conditional inference, is that the denial of the consequent is the denial of the antecedent; 'C is not D, therefore A is not B.' If the weather is fine, we go to the country; 'we are not going to the country, therefore the weather is not fine' This is still mere formal equivalence. It is implied in what has already been stated. It is not a distinct fact, but the same fact, in obverse. 'X is followed by Y' implicates one of two statements; X has happened, hence Y has followed; or,—Y has not happened,

hence X has not happened (if it did, Y would follow). Such is the two-fold bearing of a conditional proposition.

It is laid down, as part of the theory of Conditional Propositions, that the granting of the consequent does not prove the antecedent; the assertion 'C is D,' does not prove that 'A is B.' 'If he has caught the infection, he will die;' his death does not prove he has caught the infection, because there are many causes of death, besides the one mentioned. This rule, or precaution, is therefore grounded on our experience, which informs us that in nature there frequently occurs a plurality of causes. The case is parallel to the rule for the conversion of a Universal Affirmative, which depends on our knowing as a fact that in such affirmations, the predicate is not necessarily co-extensive with the subject, but is most frequently larger than the subject.

If the condition given were the sole condition of the consequent, the affirmation of the consequent would be the affirmation of the antecedent. 'If force is expended, an equivalent force will be generated' is a statement containing the one indispensable condition of the effect (an equivalent force generated). Under all possible circumstances, the production of force supposes a prior force expended: hence the affirmation of the consequent (the generation of force) is the affirmation of the antecedent (the expenditure of force). Such conditionals, however, being the exception, and not the rule, logicians forbid the affirmation of the antecedent from the affirmation of the consequent.

On the same ground it is forbidden to deny the consequent, because the antecedent is denied; A is not B, therefore C is not D; 'the man has not caught the infection, and therefore he will not die.'

The common form of conditional proposition is when both the members are affirmative. But either member, or both, may be negative. There are thus four forms:—

(1) If A is B, C is D.

(2) If A is not B, C is D. 'If the rebellion be not crushed, the king will be executed.' It is equally proper to say that the rebellion having been successful, the king's execution is certain, or that if the king is not executed, the rebellion has been crushed. 'If the jury cannot agree, they will be discharged.' If the jury be not discharged, they have agreed. 'If succour be not speedily sent, the city will surrender.' If the city does not surrender, succour has been sent.

(3) If A is B, C is not D. 'If the will of Henry VIII. was valid, James I. had no legal title to the throne of England: If James I. had a legal title, then the will of Henry was not valid.'

'If the harbour is frozen, the ships cannot come in: If the ships can come in, the harbour is not frozen.' So 'He can't be wrong whose life is in the right: If he is wrong, his life is not in the right.'

(4) If A is not B, C is not D. 'If inspectors be not appointed, no regard will be paid to the act.' This implies that if the act is observed, inspectors have been appointed. 'No Bishop, no King.' If the king is, the bishops are. 'If there be no God, no future life awaits us: If a future life does await us, there is a God.'

These forms are all regulated by the same law of transposition. The chief interest of (2) and (3) lies in this, that when both forms apply to two propositions, the union of the two is equivalent, as we shall see, to a disjunctive proposition.

32. The DISJUNCTIVE PROPOSITION may appear in the following forms:—

- I. A is either B or C.
- II. Either B or C exists.
- III. Either A is B, or C is D.

'He is either a fool or a rogue' means 'If not a fool, he is a rogue, and if not a rogue, he is a fool.' Otherwise, 'Not being a fool, he is a rogue,' and 'Not being a rogue, he is a fool.' These are all equivalent forms; and the supposed reasoning consists merely in electing one alternative, according to the facts of the case. The datum being, 'he is a not a fool,' we use the alternative 'he is a rogue,' and so on.

This corresponds to the working out of a Logical Division. 'Feelings are either pleasures, pains, or neutral excitement.' The equivalent propositions are such as these:—a feeling not a pleasure, is either pain, or a neutral state; a feeling not a pain, and not neutral, is a pleasure; a feeling not neutral is either pleasure or pain, and so forth. There is no real inference in these transmutations. They are strict equivalents of the original Disjunctive Division.

Compared with the Conditional propositions, this form exhibits a greater degree of complexity in the relation of dependence. The Conditional form expresses a simple or one-sided dependence; the presence of the first gives the presence of the second, and the absence of the second implies the absence of the first. The Disjunctive proposition indicates a double or reciprocal dependence; the presence of either is the absence of the other, and the absence of either is the presence of the other. This is the ordinary case, but the disjunctive form might be employed when the presence of either implied the presence of the other, the absence of either, the absence of the other. Thus, 'Everything in nature is

either inert or has no weight.' From this we derive the following :—

- (1) It is inert, and so it has not no-weight = it has weight.
- (2) It is not inert, and so it has no weight.
- (3) It has no weight, and so it is not inert.
- (4) It has not no-weight, i.e. it has weight, and so it is inert.

Owing to the double negation, this form is very awkward ; but it shows an intermediate stage between the conditional and the ordinary disjunctive propositions.

'You must either pay a fine or go to prison' implicates four facts :—

- (1) If you pay the fine, you don't go to prison.
- (2) If you don't pay the fine, you go to prison.
- (3) If you go to prison, you don't pay the fine.
- (4) If you don't go to prison, you pay the fine.

A disjunction is not thoroughgoing and valid unless it gives four true propositions in that form, and the only sure test of its validity is to put it through the forms. Thus :—

'Either the witness is perjured, or the prisoner is guilty,'

- (1) If the witness is perjured, the prisoner is not guilty.
- (2) If the witness is not perjured, the prisoner is guilty.
- (3) If the prisoner is guilty, the witness is not perjured.
- (4) If the prisoner is not guilty, the witness is perjured.

The propositions (2) and (4) are correct, but (1) and (3) could not be maintained. This reveals a weakness in the form of the statement. Put thus—'If the witness tells the truth, the prisoner is guilty'—the assertion is perfectly accurate, for the witness may be perjured, and still the prisoner may be guilty ; or the prisoner may be guilty, and still the witness may not have told the truth.

'Punishment is intended either to repress crime or reform the criminal.'

'If punishment represses crime, it does not reform the criminal (1).' Here we see at once that both things may concur.

'Either the ballot must be given, or intimidation will prevail.'

If intimidation does not prevail, the ballot exists (4). This would not be affirmed, and therefore the disjunction is not thoroughgoing.

'For many years past, this country has been governed either by the Whigs or by the Tories' leaves open a third case, namely, by a coalition.

'He either cannot, or will not, do it' leaves open the supposition of 'neither.'

'The substance held in solution is either lime or magnesia' is an example from chemistry, and deserves to be put through all the forms, as each form is a test.

- (1) If the reaction of lime is given, magnesia is not present.
 - (2) If the reaction of lime is not given, magnesia is present.
 - (3) If the reaction of magnesia is given, lime is absent.
 - (4) If the reaction of magnesia is not given, lime is present.
- A chemist would not be satisfied without trying two of these forms, a positive and a negative.

33. The DILEMMA combines a Conditional and a Disjunctive proposition.

If the *Antecedent* of a conditional is made disjunctive, there emerges what Whately calls a simple *Constructive Dilemma*.

If either A or B is, C is.

Now, either A or B is.

Therefore, C is.

If either plants or animals are found, there must have been previous germs.

Now, either plants or animals are found.

Whence, there have been previous germs.

The *Consequent* being made Disjunctive, gives the more usual type:—

If A is, either B or C is.

If the barometer falls, there will be either wind or rain. Various suppositions may be made, bringing out the possible alternatives. Thus—

A is; then, B or C is.

C is not; then, If A is, B is.

C is; then, If A is, B is not.

B is; then, if A is, C is not.

B is not; then if A is, C is.

B is not, and C is not; then, A is not.*

* Another form of simple Dilemma is

If B is, A is; and if C is, A is.

Now, either B or C is.

Whence, A is.

This form is illustrated by a sentence from Macaulay:—

Predestination makes men immoral; for if a man be an heir of grace, his exertions must be useless; if an heir of wrath, they must be unavailing.

If a man be an heir of grace, his exertions are useless; if of wrath, unavailing.

But, according to predestination, a man is an heir either of grace or of wrath; therefore, according to predestination, his exertions must be useless.

But he who, believes his exertions to be useless must be immoral; therefore, predestination makes men immoral.

This last is the true dilemma, which is *Destructive*. The forms preceding are equally valid, and are occasionally applicable. For instance—

If the orbit of a comet is diminished, either the comet passes through a resisting medium, or the law of gravitation is partially suspended.

" But the second alternative is inadmissible.

Hence, if the orbit of a comet is diminished, there is a resisting medium.

The conclusion is a simple conditional proposition, the complexity having been reduced.

The following are examples of the common Dilemma :—

If a classical education is worth the cost, either it must be pre-eminently fitted to develop the mental powers, or it must furnish exceedingly valuable information. But neither alternative can be maintained, and so a classical education is not worth the cost.

If schoolmasters can claim exemption from poor's rates, it must be either by statute or by the common law. Now, no statute exempts them; and the common law does not apply. Hence they can claim no exemption from Poor's Rates.

Sometimes the antecedent is more conveniently put in the form of a question.

How do we know that our intuitive beliefs concerning the world are invariably true? Either it must be from experience establishing the harmony, or an intuitive belief must certify the correctness.

Now, experience cannot warrant such harmony except in so far as it has been perceived. Still more futile is it to make one instinctive belief the guarantee of another. Thus we cannot know that any intuitive belief is universally valid.

The Dilemma, although occasionally a useful form, is perhaps oftener a snare. The point is whether the disjunction is valid; and there is always supposed the rejection of many possible cases. We begin with—If A is, B or C or D or E is. One after another of the suppositions is rejected, until at last only two are left, and these being removed, the antecedent is finally denied. The illusive case is when the logician trusts to the law of excluded middle as a guarantee of the disjunction. If A is, A is either B or not-B. We may easily affirm that A is not B, but how can we affirm that it is not not-B, i.e. it is neither B nor anything else than B. It is plain that if we were able to affirm that A is not anything else than B, we should not

require a dilemma nor yet the term B to disprove A's existence. As an example of a false disjunction, we may take the ancient fallacy of Motion.

If a body moves, it must move either in the place where it is, or in the place where it is not.

But a body cannot move in the place where it is, nor yet in the place where it is not. Hence, a body cannot move at all!

The disjunction to conform to the law of Excluded Middle must be in this form:—

The body must move in the place where it is, or it must *not* move, in the place where it is. We then admit that a body does not move *in* the place where it is, and the possibility of motion is still undestroyed.

'If the books in the Alexandrine Library be in conformity with the doctrines of the Koran, there is no need of them, if they are adverse to the doctrines of the Koran, they should be destroyed.' This is not exhaustive, as the books might not treat of religion; but the assertion implies that no knowledge is desirable except religious knowledge.

'A Berkeleian is reduced, in truth, to this dilemma: if he knows what external things are, it can only be by perceiving them as external,—which contradicts his theory. If, on the other hand, he does not know what they are, he is incapable of using the expression *external* with any meaning, and could, in fact, never have invented or thought of employing it.' This assumes that the meaning of 'external objects' is not in dispute; it is a summary mode of stating one side; Berkeley could say that the meaning of external objects was just the point in dispute.

. . *Synonymous Propositions.*

34. Every language contains various wordings for the same matter of fact; and there is occasionally an advantage in passing from one of these to the other. We may call these variations *Synonymous Propositions*.

There being, in many instances, a plurality of names for the same object, or the same fact, we find them freely interchanged. The essential characteristic of all material substance is expressed as Resistance, Force, Momentum, Inertness, all which mean the same thing, although viewed in different aspects.

'Men are mortal,' 'all will die,' 'we are doomed to dissolution,' 'decay is the law of our being'—are mere synonymous

variations that add nothing to the fact, but may contribute to the force of it.

'This weighs that down, therefore, it is heavier,' is not a real inference; the two expressions signify one operation. There is no other criterion of the comparative heaviness of two things, but weighing them. This block of marble is *larger* than that, therefore, it is *heavier*, is a real inference. The superior size is given as the evidence of superiority in another and different quality, weight.

'What has been, will be;' 'the future will resemble the past;' 'nature is uniform;' 'the laws of the universe are constant;'—these are all synonymous expressions for the same fundamental fact. One of them cannot be tendered as the reason or evidence of another. The multiplication of forms may aid in expounding the great truth underlying them all. One form may be suggestive of one class of examples, a different form may suggest another class. The variation of language is often a great intellectual help. It is, however, a source of danger. One of the lures and snares of language lies in the tendency of the mind to suppose that two different forms of expression mean two different things. Hence, it is a common fallacy, and a device of Rhetoric, to give a fact as the reason for itself; there being merely a change in the expression.

There is often a difficulty in finding a single satisfactory expression for notions and truths of great generality. The great law of the Conservation of Force, needs the aid of other terms to suggest all its meaning—Persistence, Exchangeability, Equivalence, Correlation. The grounds of the Transcendental part of Algebra, called the Differential Calculus, have been viewed in a great variety of aspects, expressed by different names—Exhaustions, Limits, Prime and Ultimate Ratios, Evanescent Quantities, Fluxions, Differential Co-efficients.

The elements of the mind called intuitive by the *a priori* school of philosophy, are stated sometimes under the guise of the Notion, and sometimes under the guise of the Proposition; the subject matter being identical. We may say either 'Cause' is an *innate notion*; or 'every effect must have a cause' is an *innate proposition*, principle, or judgment.

The Dictionary mode of defining words consists in giving tautologous phrases, which shows that these abound in language. If there were only one name for one thing, an English Dictionary, conceived on the usual plan, could not exist.

EXERCISES ON PROPOSITIONS, INCLUDING NOTIONS.

The following are examples of Propositions, to be used as exercises, in connexion with the Classification of Propositions, and the Equivalent Forms. As every real proposition has two notions, while even verbal propositions contain at least one notion, the examples will also furnish exercises on the Notion.

As regards the Class or Notion, in opposition to the Real Proposition, the points to be illustrated are comparatively few. An Individual or Singular object or thing may be exhibited in contrast to classes or Generalities; Homer to poets, the Rhine to rivers; Britain to sovereign states. Of generalized things, we have the Class (concrete), and the Attributes (abstract). The *grades* of generality may be exemplified,—a very valuable exercise. There remains only the illustration of Relativity, the assigning of the correlative class or notion in a definite universe.

The Notion often condenses in a word what would require one or more propositions to express in full. Refraction, Electricity, Crystallization, Chemical Affinity,—are names for complex facts, involving many propositions, and not to be explained without giving these propositions. 'Refraction' is the summary designation of the principle or law of the bending of light in passing from one transparent medium to another; and its full and proper expression is the law itself given as a real predication.

The various aspects of the Proposition, exhibited in the foregoing chapter, may be summarized as follows:—

I. AS INDIVIDUAL or GENERAL, and as of different *grades* of Generality, under which is brought out the diminishing Connotation or Comprehension that accompanies increasing Generality or Extension.

The principle of Relativity applied to Propositions, appears under various subsequent heads—Negation, Opposition, and Obversion.

II. AS POSSESSING QUANTITY and QUALITY, with reference to the uses of Syllogism.

III. AS COMPLEX in contrast to SIMPLE; the important logical example of Complexity being HYPOTHETICAL propositions (Conditional and Disjunctive).

IV. AS *opposed* in the various modes named CONTRARIES, CONTRADICTORIES, &c.

V. As in their final Import, affirming EQUALITY, Co-EXISTENCE or SUCCESSION; the two last containing the special kinds named respectively *Co-inhering Attributes* and *Causation*.

In this connexion, there might be given the particular *Science* that the proposition belongs to:—as Mathematics, Chemistry, Psychology, &c. For although propositions of Equality make up the one science, Mathematics; those under the two other heads—Co-inhering Attributes and Causation—are distributed among several sciences.

VI. As having numerous EQUIVALENT FORMS, namely General and Particular, Greater and Less in Connnotation, Obverse, Converse, Hypothetical Equivalents, Synonyms.

VII. All the foregoing classes suppose real predication. It is, however, important to taking every opportunity of contrasting REAL with VERBAL propositions. A farther interest attaches to the difference between predicating a *Proprium* and predicating a *Concomitant*.

Many of the propositions occurring in common speech are not certain, but only *probable*; the affirmation holds not in all cases, but in a very great number, as ‘Temperate persons are long lived.’ The subject of Probability belongs to the Inductive Logic, and has not been adverted to in the foregoing classification. Still, the distinction of probable and certain is so easily understood, in the main circumstance, and so important to be born in mind, in matters of truth and falsehood, that it should be impressed on every suitable opportunity.

At the present stage, consideration is given, not to the actual truth and falsehood of propositions, but only to what they profess. The proof or evidence of assertions belongs to the subsequent heads—Deduction and Induction.

Of the following examples, promiscuously chosen, the various forms are to be used according to their peculiar suitability for the different classes of propositions. In a large proportion of them, there is scope for translating the idioms of ordinary language into modes of expression more in accordance with the logical forms.

‘Honesty is the best policy.’

A proposition of a certain grade of Generality; one relating to ‘virtue’ would be more general; one relating to ‘paying one’s debts’ would be less general, but would have a more comprehensive predicate.

As regards Quantity and Quality (in Form), it is a universal

affirmative; being translatable into 'all honest actions are more politic than actions not honest.'

We read, in Otway, 'Honesty is a damned starving quality,' which is the full Contrary. The Contradictory is, 'Some honest actions are not good policy.'

In Import, the proposition is one of Causation—'Honest actions bring good consequences to the agent.' The subject being Mind, it belongs to the science of Psychology.

Many Equivalent Forms could be given—'Some honest actions are politic.' Obversion (Formal):—'Honesty is not bad policy;' 'No honest men are unsuccessful men;' (Material) 'Dishonesty is bad policy.' Conversion:—'Some politic actions are honest actions.'

The proposition is not verbal but Real; good policy is not, in whole or in part, the definition of honesty. It is a Proprium, or derivative proposition, and not an ultimate fact; it is deducible from the operation of honesty, under general laws of cause and effect in the human mind.

It is a proposition, not certain, but Probable. It is true, not universally, but in a large and preponderating number of cases.

'All the alkalis and alkaline earths are oxides of the metals.' A complex affirmation, containing two in one, which must be taken separately. In form and import, they are so closely allied, that one may represent both.

As regards External Form, each is an example of A, with no peculiarities requiring attention.

In Import, they belong to the class of affirmations of Co-inhering Attributes, and fall under Chemistry.

Strictly analyzed, each is a verbal proposition; the predicate—oxides of the metals—is now given as one of the essential characters of Alkalis, and of Alkaline Earths. In the original connotation of these words, however, the composition or derivation of the substances was not taken into account; the main fact was the relation to acids, and to neutral salts. At that stage, Davy's discovery was an additional fact, and therefore a real predication. In so far as the terms still suggest to the mind only the primitive meaning of an Alkali, the proposition is, not essential and verbal, but real.

'Fishes breathe by gills.' Equivalent to 'All fishes.' A verbal or essential proposition of Kinds; the subject 'fishes' connotes all the essential attributes of fishes, of which the pre-

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sent is one. As the structure is confined to fishes, the subject and predicate are co-extensive. It is a proposition in Biology, or Zoology.

‘One aid to health is exercise.’ An inversion for—‘Exercise aids or promotes health.’ ‘All persons that take exercise use one of the aids to health.’ A proposition of Cause and Effect, in Biology. A Real proposition.

‘Pain is a consequence of Sensibility.’ (Concrete) All sensitive being are beings subject to pain; all sensitive beings, under certain circumstances, are pained beings. A Verbal or analytical proposition; ‘being subject to pleasure, to pain and to neutral excitement,’ is the definition of ‘Sensitive.’ Might be given to illustrate the Aristotelian distinction of the *Potential* and the *Actual*.

‘Whatever is, is right.’ The generality of the subject is even beyond the two *summa genera*—Object and Subject. Existence is a fictitious predicate, and, in intelligible propositions, means something more definite than it seems. The proposition must be interpreted—‘all the arrangements of the world are right, or are good.’ In Import, this is Cause and Effect. The obverse is ‘nothing that is, is wrong,’ ‘there is no wrong.’

‘The Beautiful and the Useful are partially coincident;’ a synonymous form for—Some Beautiful things are useful, and conversely.

‘The wages of sin is death,’ or Death is the wages of sin. This form would suggest a universal co-existence between Death and Sin—all beings that die are all beings that sin. Another interpretation is ‘Adam’s sin was the cause of death.’

‘Self-confidence is not inconsistent with great weakness.’ ‘Self-confident persons may be weak persons.’ This is a contradictory to ‘All self-confident persons are strong.’

Of a similar nature is—‘A proud man is not necessarily a bad man.’

‘Man is the only animal combining sociability and solitude. A form equivalent to the universal Quantification of the Predi-

cate, and useful to test De Morgan's criticism as to the denial of such propositions.

Take together the 47th and the 48th propositions of the First Book of Euclid, and show their bearing on universal quantification.

'Adverbs qualify verbs;' 'Adverbs are to be placed near the words they qualify.' How do these differ logically?

'The greater the novelty, the greater the pleasure.' A proprium or inference from 'Novelty is a source of pleasure.' In propositions of cause and effect, we are entitled to infer the proportionality of the one to the other.

'Symmetry is the general law of creation;' a greatly distorted expression of what is meant. 'Symmetry' is a word condensing a proposition; and the sounding phrase 'the general law of creation' signifies merely that a fact is frequent or usual. 'Many (or some) things in nature are symmetrically constructed.'

The angle in a semicircle is a right angle.

Ice is cold.

The diamond is surpassingly brilliant

Extreme heat destroys life.

Motion follows the line of least resistance.

Truth is more easily extricated from error than from confusion.

An age of ignorance is an age of ceremony.

Power corrupts the mind.

Time abates grief.

Custom blunts sensibility.

Private vices are public benefits.

Uneasy lies the head that wears a crown.

Tyranny is irresponsible power.

Benevolence is the sum of virtue.

Distance lends enchantment to the view.

Consumption is a fatal disease in this country.

International law has no written statutes.

Conception is involved in every act of perception.

None but the brave deserve the fair.

Not being rich is not always an evil.

All is not gold that glitters.

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The causes of strength are not pledges for its continuance.

Not every advice is a safe one.

A great deal need not be attempted.

He is no fool.

No news is good news.

• No men are placed in exalted situations and free from envious regards.

Good orators are not always good statesmen.

There are studies much vaunted, and yet of little utility.

Few even of our best aspirations are gratified.

Hardly any virtue is quite safe from passing into a vice.

The two following extracts are from Plato—

‘All men who have gout, or fever, or ophthalmia, are sick ; but all sick men have not gout, or fever, or ophthalmia. So, too, all carpenters, or shoemakers, or sculptors, are craftsmen ; but all craftsmen are not carpenters, or shoemakers, or sculptors. In like manner, all madmen are unwise ; but all unwise men are not mad.

‘Whosoever is a good rhapsode, is also a good general ? Unquestionably. And, of course, whoever is a good general, is also a good rhapsode ? No ; I do not think that.’

‘The objects bring up the feelings, and, *conversely*, the feelings the objects.’ In this sentence, is the word ‘conversely’ used in its proper meaning ?

It steam is passed over red hot iron, hydrogen will be evolved.

If virtue is knowledge, it is teachable.

If the footmarks were made by the prisoner, he must have worn shoes too small for his feet. But he could not have done so. What then ?

If the soul is incorruptible, it is ingenerable.

Matter is either solid, or liquid, or gaseous.

Mr. de Morgan supposes a stump orator intending to say—*all* Englishmen are lovers of liberty ; and declaiming in these terms :—‘Shew me any number of men, and I will say with confidence, either that they will with one accord raise their voices for liberty, or that there are aliens among them.’ This might be regarded as an equivalent statement, without syllogistic inference.

Cromwell, on his death-bed, is said to have asked a divine

who was with him, whether it was possible to fall away from grace. The answer was,—It is not possible. Then, said Cromwell, I am safe, for I was in grace once.

. No form of polity is so admirable as a limited constitutional monarchy; for it is, beyond all question, superior to every other species of government.

• Honesty is deserving of reward. A negro is a fellow creature. An honest negro is a fellow-creature deserving reward.

Every man is an animal. Every head of a man is the head of an animal. De Morgan.

In Book IV—The Logic of the Sciences—as well as throughout the work generally, there occur numerous examples that may serve as additional exercises if necessary.

BOOK II

DEDUCTION.

CHAPTER I

THE SYLLOGISM.

1. The SYLLOGISM is the fully expressed form of a Deductive Inference, that is, an inference from the General to the Particular.

When a step of reasoning or argumentation consists in assigning, as the proof of an affirmation (or denial), some more general affirmation, it admits of being stated in a peculiar form, in which there is sometimes greater facility in judging of its soundness. The peculiarity of the form of statement consists mainly in this, that everything belonging to the reasoning is set forth explicitly. Thus, when any one maintains that Mathematics is useful as a mental discipline, and assigns as the proof, that all the exact sciences are useful as mental discipline, the reasoning, which is Deductive, and not Inductive, contains these two assertions:—(1) All the exact sciences are useful as mental discipline; (2) Mathematics is an exact science. Both these are indispensable to the conclusion ‘Mathematics is a mental discipline.’ The first is the general principle, the second an intermediate proposition for applying the general principle to the case in hand. Very often, one of the two propositions is left unexpressed. In the example: ‘this man is a rogue, therefore he is not to be trusted,’ there is an ellipsis of the general principle—‘rogues are not to be trusted.’ In the form ‘you cannot trust rogues, therefore you cannot trust this man,’ the omission is in the second or applying proposition—‘this man is a rogue.’

A Deductive reasoning fully and formally expressed is a Syllogism.

The following arrangement—

- (1) All men are fallible,
- (2) John is a man,
- (3) John is fallible—

is a regular deductive reasoning, or an argumentation in the syllogistic or complete form. The two first propositions combine to make the proof of the third; they are called the *Premises* of the reasoning or syllogism; the third is the point to be proved, and is called the *Conclusion*.

We shall see hereafter that, in the departures made from the regular form of the syllogism, the order of the propositions may be reversed; the applying proposition coming first, and the grounding proposition second. But whatever form the syllogism may assume, one feature can never be absent—a general proposition. This is indispensable. Unless one of the premises be more general than the conclusion, the argument is not deductive.

2. A Syllogism is said to contain three, and only three Terms; the Subject and the Predicate of the Conclusion, and another Term, occurring in both Premises; the Subject of the Conclusion is the *Minor* Term; the Predicate of the Conclusion, the *Major* Term; the term occurring in both Premises, is the *Middle* Term.

By 'Terms' are meant the expressed notions entering into the subjects and predicates of the propositions. A proposition couples or unites two Terms. 'X is Y' contains the two terms X and Y affirmatively conjoined. 'Men are not gods' contains the two terms 'men' and 'gods' under a negative copula.

In seeking out the Terms, we begin with the proposition to be proved, that is, the conclusion. The *subject* of the conclusion is the Minor or smaller term, the *predicate* the Major or greater term. The propriety of these designations is grounded on the circumstance, formerly adverted to, that in propositions generally, the predicate covers the subject, and other subjects besides; 'kings are fallible,' and many other beings besides kings are fallible; hence 'kings' are a smaller group forming part of a larger group 'fallible;' in compass or extent, therefore, 'kings' are a *Minor term*, 'fallible' a *Major term*.*

* Sir W. Hamilton complains that these designations are false and erroneous because they do not apply to the terms as considered in Comprehension. There are more men than kings, and so the designations are

The *Middle Term* must be sought not in the conclusion, but in the Premises, or proving propositions, and must appear in both. Thus, in the syllogism—

Men are fallible,
Kings are *men*,
Kings are fallible.

The term, absent from the conclusion, and present in both premises, is 'men,' the subject of the first and the predicate of the second. It is called 'middle' because it is the medium or instrumentality for bringing together in the conclusion, the major and minor terms; they being separated in the premises. Also, as regards extent, compass, or denotation, it is intermediate thus:—The minor 'kings' is less in extent than 'men'; men are more numerous than kings. Again, 'men' is less in extent than 'fallible beings'; there being many fallible beings besides men. So 'men' being more extensive than the minor term 'kings,' and less extensive than the major term 'fallible beings,' is properly a middle or intermediate term. The gradation is represented in a diagram thus:—

| | | | | |
|-----------|---|---|---|----------------|
| Fallible, | . | . | . | <i>major,</i> |
| Men, . | . | . | . | <i>middle,</i> |
| Kings, | . | . | . | <i>minor.</i> |

Although the syllogism contains three propositions, each with two terms, making six terms in all; yet, in virtue of the double occurrence of each, there are in reality only three terms. The example shows:—

The *Middle* term in *both* premises.

The *Minor* term in the conclusion and in one premise.

The *Major* term in the conclusion and in one premise.

3. In the Syllogism, there are Three, and only three, Propositions, namely, the two Premises and the Conclusion. The Premise containing the Major Term and the Middle Term, is called the *Major Premise*; the Premise containing the Middle Term and the Minor Term, is called the *Minor Premise*.

In the foregoing example, the Premise first in order contains applicable to the extension of the terms; but, he argues, more attributes are connoted by the term 'kings' than by the term 'men,' and so major and minor are inapplicable to the comprehension. In criticism of this view, it may be said that confessedly the designations *major* and *minor* are applicable to the terms viewed in their compass or extension, that these terms are used in that sense, that they cannot be used without confusion in both senses, and that Hamilton has shown no good reason for inverting the common usage.

the Major term 'fallible,' together with the Middle term, 'men,'—'men are fallible;' this is the *Major Premise*. The Premise second in order contains the Middle term, 'men,' and the Minor term, 'kings,'—'kings are men'—and is the *Minor Premise*.

We find it convenient to represent the forms of the syllogism by letters or symbols, thus :—Let X be the minor term, Y, the middle term, Z, the major term; then—

All Y is Z

All X is Y

All X is Z

is a syllogistic form on the basis of affirmation; that is to say, the universal proposition in the first premise is affirmative, and the conclusion is affirmative.

An example on the basis of negation is—

No Y is Z

All X is Y

No X is Z,

or, by Hamilton's still more expressive symbols,—

S (subject of conclusion, *minor* term),

M (*middle* term),

P (predicate of conclusion, *major* term);

All M is P No M is P

All S is M All S is M

All S is P No S is P.

4. Syllogisms, or Syllogistic forms, are divided into FIGURES, according to the position of the Middle Term. There are, in all, Four Figures.

The First Figure is exemplified in the forms hitherto employed. In it, the Middle Term is *Subject* in the Major Premise, *Predicate* in the Minor Premise.

Y is Z M is P M —

X is Y S is M — M

X is Z S is P

The idea implied under 'Figure' is borrowed from the Figures of Rhetoric, which are departures, for effect, from the plain and ordinary forms of speech. On this analogy, however, as remarked by Hamilton, there ought to be some one *regular* or *standard* form, from which all other forms are deviations or departures, thence properly called 'Figures.' Such standard form is what is mis-named the 'First Figure,' which is the pure type of a deductive argument. The Major or First Premise is the universal proposi on indispensable in

deduction, the Minor or Second Premise is an affirmative proposition, whatever may be its quantity. As to order, the Universal is placed first, as being of the two premises the fundamental or chief; the use of the second premise, the minor, being to apply the first to a particular case. 'All thieves are deserving of punishment,' is applied to a particular instance, by means of an affirmation bringing the instance within the sweep of the rule, that is, declaring such a one to be a thief. This is the function of the minor.

In the Second Figure, or the first departure from the normal syllogism, the middle term is predicate in both premises

| | | |
|--------|--------|-----|
| Z is Y | P is M | M |
| X is Y | S is M | — M |

Here there is an obvious inversion of the natural order of things. In the major premise, Z is Y, P is M, the largest term is made the subject, and the middle term the predicate, of the proposition. If the proposition be affirmative, this change is not compatible with universality, and therefore the proposition cannot be the major in the same sense as in the standard syllogism. If the proposition be negative, there is only a harmless conversion; we may, for 'no Y is Z,' substitute 'no Z is Y;' 'no men are gods' 'no gods are men.' This is an insignificant and, for the most part, useless alteration of the negative form of the standard syllogism. Two of the four forms of the Figure (called *Moods*) are fashioned out of this trivial alteration. The two other forms containing affirmative majors involve still greater changes of the standard form. In one, the major is not the universal proposition required as the basis of the deduction, but the applying proposition, which in the first figure is the second or minor premise. In the concluding form, there is a much greater distortion, consequent on presenting the normal premises in obverted forms.

In the Third Figure, the middle term is subject in both premises.

| | | |
|--------|--------|----|
| Y is Z | M is P | M— |
| X is X | S is S | M— |

Here the major stands as in the first, or normal figure. The minor has its terms transposed; the middle term is subject, and the minor term predicate. As before, this is a harmless change, if the proposition be a universal negative; in which case, however, the minor premise must be the universal or grounding proposition, and not the applying proposition; so that, as compared with the standard form, there is an inversion of the order of the premises. If the minor be affirmative, either it

must be particular, or there is some distortion, rendering the terms different in fact from what they are in appearance.

In the Fourth figure, the position of the middle term is the first figure reversed; it is predicate in major, and subject in minor.

Z is Y P is M — M
Y is X M is S M —

This double inversion of the order of the terms implies still greater deviations from the primary form. The inversion is possible by such devices as above described for the smaller inversions in the second and third figures.

5. Each Figure has a certain number of distinct forms, called the *Moods*, or modes of the figure. The variation of mood is determined by the variety of the propositions contained, as regards Quantity, and Quality.

The order of the terms is fixed for each Figure; but the propositions constituting the premises and the conclusion may, within certain limits, be of one or other of the four forms, A, I, E, O.

The First Figure, the normal syllogism, has Four Moods.

The First Mood is composed of three universal affirmations.

| | | |
|------------|-----------|-------------------------|
| All Y is Z | A, A, A | All men are fallible. |
| All X is Y | (Barbara) | All kings are men. |
| All X is Z | | All kings are fallible. |

In the Second Mood,

| | | |
|----------------|-------------------------|--------------------|
| The Major is | a universal negative | —E. |
| The Minor | a universal affirmative | —A. |
| The Conclusion | a universal negative | —E. |
| No Y is Z | E, A, E | No men are gods. |
| All X is Y | (Celarent) | All kings are men. |
| No X is Z | | No kings are gods. |

The Third Mood is the first, with a particular minor, and particular conclusion:—

| | | |
|-------------|---------|---------------------------|
| All Y is Z | A, I, I | All men are fallible. |
| Some X is Y | (Darii) | Some beings are men. |
| Some X is Z | | Some beings are fallible. |

The Fourth Mood is a similar variation on the second; particular minor and particular conclusion:—

| | | |
|-----------------|---------|---------------------------|
| No Y is Z | E, I, O | No men are gods. |
| Some X is Y | (Ferio) | Some beings are men. |
| Some X is not Z | | Some beings are not gods. |

These four moods are obviously reducible to two; the third and fourth being mere unessential varieties of the first and second. The two comprehensive forms may be stated thus:—

| | |
|--------------------|--------------------|
| All Y is Z | No Y is Z |
| All or some X is Y | All or some X is Y |
| All or some X is Z | No X is Z. |
| | Some X is not Z. |

The first form is the normal type of all deduction for an affirmative conclusion; the second, the type for a negative conclusion. They present the deductive process in its regular order:—

First, a universal proposition, as the ground proposition of the reasoning (Major premise);

Secondly, an affirmative and applying proposition (Minor premise);

Lastly, the universal truth applied to the particular case (the Conclusion).

We desire to prove that kings are fallible, by applying to them the principle of the fallibility of all men. The major states the principle, the minor applies it. And so for a negative conclusion.

There cannot be any valid deduction whatsoever but must conform to the foregoing type; whatever variation may be made, this is at the bottom.

The SECOND FIGURE has likewise four Moods.

In the *First Mood*,

The Major is a universal negative —E.

The Minor a universal affirmative—A.

The Conclusion a universal negative —E.

No Z is Y } E, A, E, No gods are men.

All X is Y } (Cesare) All kings are men.

No X is Z } No kings are gods.

This is a case where advantage is taken of the simple conversion of the universal negative to make a trivial departure from the standard (negative) syllogism. Only a slight change is necessary to reconvert the present mood to the second mood of the First Figure; for 'No Y is Z' 'No men are gods,' we are at liberty to substitute 'No Z is Y,' 'No gods are men,' which is the whole difference.

In the *Second Mood*,

The Major is a universal affirmative—A,

The Minor a universal negative—E,

The conclusion a universal negative—E.
 All Z is Y } A, E, E, All kings are men.
 No X is Y } (*Camestres*) No gods are men.
 No X is Z } No gods are kings.

A much greater variation from the standard (negative) is observable here. The grounding proposition, which must be universal, is the minor premise: so that there is an inversion of the normal order of the premises. Moreover, the same proposition has been converted simply, from the form 'No men are gods;' and the conclusion is likewise the converse of the conclusion in the regular syllogism. By first restoring the order of the premises, and next re-converting two universal negations, we have the normal negative syllogism (*Celarent*).

No men are gods.
 All kings are men.
 No kings are gods.

The grounding universal is the negative proposition, 'no men are gods'—the applying proposition is 'all kings are men.'

In the *Third Mood*.

The Major is a universal negative —E,
 The Minor a particular affirmative—I,
 The Conclusion a particular negative —O.
 No Z is Y } E, I, O No gods are men.
 Some X is Y } (*Festino*) Some beings are men.
 Some X is not Z } Some beings are not gods.

Here we remark the same trivial departure from one of the standard forms, as in the first mood. The universal negative—the major in the fourth mood of the first figure (*Ferio*)—is simply converted (No Y is Z, into No Z is Y; no men are gods, into no gods are men).

In the *Fourth* and last *Mood*, there is a more serious distortion.

The Major is a universal affirmative—A,
 The Minor a particular negative —O,
 The Conclusion a particular negative —O.
 All Z is Y } A, O, O All gods are men.
 Some X is not Y } (*Baroko*) Some beings are not men.
 Some X is not Z } Some beings are not gods.

A glance at the premises shows us that they are not at bottom what they appear on the surface. There is indeed a universal proposition in the major premise, which might answer for the ground proposition; but then the other pre-

mise, in that case the applying proposition, is negative, which is not allowable. The real fact is that the affirmative major, is a negative (universal) in disguise, and the negative minor, is an affirmative in disguise. The disguises may be laid open, thus—

| | |
|-----------------|-----------------|
| All Z is Y | No not-Y is Z |
| Some X is not Y | Some X is not-Y |
| Some X is not Z | Some X is not Z |

The true middle term instead of being Y, is the negative of Y, or not-Y (U—Y). This is the key to the distortion. The remedy consists in (1) *obverting* and *converting* the major—All Z is Y, which becomes No not-Y is Z; and (2) in *obverting* the minor—Some X is not Y, Some X is not-Y. There thus emerges a form of the third mood of the first figure (*Ferio*), with not-Y, as the middle term.

This mood cannot be reduced to a mood of the First Figure without Obversion. The older logicians sought to establish its validity by a cumbrous process technically known as *Reductio ad impossibile*. They showed that the conclusion cannot be supposed false, without leading to a contradiction of one of the premises, which are given as unimpeachable. Thus:—

All Z is Y
Some X is not Y
Some X is not Z

If 'Some X is not Z' be declared false, the universal 'All X is Z,'—which is its contradictory,—must be admitted as true. Taking this new proposition, 'All X is Z' along with the major of the original syllogism, 'All Z is Y,' we reach the conclusion that 'All X is Y.' Thus:—

All Z is Y
All X is Z
∴ All X is Y

is a syllogism in *Barbara*. But we know from the original premises that 'Some X is not Y;' it cannot therefore be true that 'All X is Y.' One of the premises of the above *Barbara* must be unsound. The major 'All Z is Y,' is one of the original premises, granted as true; the error must lie on the minor, 'All X is Z.' Now this is the proposition taken on trial; and its truth being shown to be incompatible with the truth of the original premises, its contradictory, 'Some X is not Z' must be true. And 'Some X is not Z' is the conclusion in question; which is thus shown to be valid.

The THIRD FIGURE has six Moods.

In the *First Mood*,

The Major is a universal affirmative—A.

The Minor a universal affirmative—A.

The Conclusion a particular affirmative—I.

All Y is Z } A, A, I All men are fallible.

All Y is X } (*Darapti*) All men are living beings.

Some X is Z } Some living beings are fallible.

The only departure, in this instance, from the standard syllogism (with a particular minor, *Darii*) is the universality of the minor, All Y is X. By simple conversion, this premise becomes Some X is Y, and the syllogism is then the same as the third mood of the regular syllogism.

This figure is quoted as a useful form. Certain reasonings are considered to fall more readily into the above arrangement, than into the corresponding mood of the First Figure.

The *Second Mood* contains an inversion of the order of the Premises. This distortion is altogether gratuitous; it serves no purpose but to seem a variety.

Some Y is Z } I, A, I Some men are kings.

All Y is X } (*Disamis*) All men are fallible beings.

Some X is Z } Some fallible beings are kings.

Here, if we redress the order of the premises, and simply convert the new minor—Some Y is Z, into Some Z is Y,—there arises a regular affirmative syllogism, with a particular minor (*Darii*); there being only the speciality that the minor and the major terms have changed places, thus:—

All Y is X All men are fallible beings.

Some Z is Y Some kings are men.

From this the conclusion would be 'Some Z is X,' 'some kings are fallible beings,' which, however, 'by simple conversion, gives 'Some X is Z,' 'some fallible beings are men.'

The *Third Mood* is one of the trivial variations of syllogistic form.

All Y is Z } A, I, I All men are fallible.

Some Y is X } (*Datisi*), Some men are kings.

Some X is Z } Some kings are fallible beings.

There is no departure here, from the regular syllogism (affirmative, with particular minor *Darii*), but in the minor premise, which is Some Y is X, instead of its equivalent, Some X is Y.

The *Fourth Mood* is exactly the counterpart of the previous mood, with a negative major.

| | | |
|-----------------|-----------|--|
| No Y is Z | } E, A, O | No men are gods. |
| All Y is X | | (<i>Felapton</i>) All men are living beings. |
| Some X is not Z | | Some living beings are not gods. |

This differs from the negative mood of the first figure, with a particular minor (*Ferio*), only in having a universal minor, which, by conversion, becomes particular, Some X is Y; the syllogism is then exactly the fourth mood of the standard syllogism.

The *Fifth Mood* is, in point of distortion, the parallel of the last mood of the Second Figure (*Baroko*). Both the premises appear different from what they are in reality.

| | | |
|-----------------|------------|--|
| Some Y is not Z | } O, A, O, | Some men are not kings. |
| All Y is X | | (<i>Bokardo</i>) All men are fallible. |
| Some X is not Z | | Some fallible beings are not kings. |

If we look for a universal premise, to supply the ground proposition, we seem to find it in the minor; but then the other premise is negative, and therefore is not the applying proposition. As in *Baroko*, we must transfigure both premises. The present major is made affirmative, by obversion,—‘Some Y is not-Z,’ and is then converted, ‘Some not-Z is Y.’ This is taken as the minor premise, the other being the major, thus:—

All Y is X All men are fallible.

Some not-Z is Y Some not-kings are men.

which are the premises of the regular syllogism (affirmative, with particular minor, *Darii*) and would give as a conclusion,

Some not-Z is X, Some not-kings are fallible,
or, by conversion and obversion,

Some X is not Z, Some fallible beings are not kings.

As in the case of *Baroko*, the older logicians could not refer this mood to the First Figure, and applied as a test of its validity the *Reductio ad impossibile*. The process need not be repeated at length. We assume the universal contrary to the conclusion, and taking it along with the given minor, evolve a proposition that contradicts the given major: and argue, as under *Baroko*, that the universal contrary of the conclusion must be false, and therefore the conclusion itself valid.

The *Sixth* and last *Mood* is the negative counterpart of the third, and should have been placed after the fourth; it is an equally trivial departure from the regular syllogism (negative, with particular premise, *Ferio*).

| | | |
|-----------------|--------------------|----------------------------------|
| No Y is Z | } E, I, O, (Ferio) | No men are gods. |
| Some Y is X | | Some men are living beings. |
| Some X is not Z | | Some living beings are not gods. |

The simple conversion of the minor 'Some Y is X,' into 'Some X is Y,' 'some living beings are men,'—reproduces *Ferio*, in the standard figure.

The **FOURTH FIGURE** has five Moods. In this figure, there is an inversion of both premises as compared with the regular syllogism. This, of course, produces apparently a great degree of distortion; but there is very little in reality. In three of the moods, the inversion is caused by the transposition of the premises; this rectified, they need only the simple conversion of one or more of the propositions to make them standard syllogisms.

Thus, to take the *First Mood*, which has universal affirmative premises, and particular conclusion:—

| | | |
|-------------|-----------------------|---------------------------------|
| All Z is Y | } A, A, I (Bramantip) | All kings are men. |
| All Y is X | | All men are fallible. |
| Some X is Z | | Some fallible beings are kings. |

Transpose the premises, and there emerges a standard syllogism (affirmative, with universal minor, *Barbara*)—

All Y is X All men are fallible.

All Z is Y All kings are men.

The conclusion from these premises is—

All Z is X All kings are fallible.

This conclusion, converted by limitation, gives—

Some X is Z Some fallible beings are kings.

The *Second Mood* is, if possible, still closer to a regular syllogism, when the order of the premises is changed.

| | | |
|------------|----------------------|--------------------|
| All Z is Y | } A, E, E, (Camenes) | All kings are men. |
| No Y is X | | No men are gods. |
| No X is Z | | No gods are kings. |

Restore the order of the Premises:—

No Y is X No men are gods.

All Z is Y All kings are men.

These are the premises of the regular syllogism (negative, with universal minor, *Celarent*), and the conclusion is

No Z is X No kings are gods.

Whence No X is Z No gods are kings.

The *Third Mood* is constructed on a similar plan; the deviation from regularity being caused by transposed premises:—

Some Z is Y } I, A, I Some living beings are men.
 All Y is X } (*Demaris*) All men are fallible.
 Some X is Z } Some fallible objects are living beings
 With re-transposed premises,—

All Y is X All men are fallible.
 Some Z is Y Some living beings are men.
 Whence by *Darii*, in the standard Figure, the conclusion is,—
 Some Z is X Some living beings are fallible.
 Or Some X is Z Some fallible objects are living beings.

The fourth and fifth Moods attain their peculiar form, not through the inverted order, but through the conversion, of the Premises. The *Fourth* runs thus.—

No Z is Y } E, A, O No gods are men.
 All Y is X } (*Fesapo*) All men are living beings.
 Some X is not Z } Some living beings are not gods.
 Convert both premises, the major simply, the minor by limitation :—

No Y is Z No men are gods.
 Some X is Y Some living beings are men.

These are the premises of the negative form in the first figure, with particular minor (*Ferio*), whence

Some X is not Z Some living beings are not gods.

The *Fifth* and last Mood differs from the fourth only in having a particular minor; the universality of the minor in the fourth being superfluous, as leading to no stronger conclusion than the present form. The process of assimilation to *Ferio* is precisely the same—

No Z is Y } E, I, O, No gods are men.
 Some Y is X } (*Fresison*) Some men are living beings.
 Some X is not Z } Some living beings are not gods.
 Convert both premises simply :—

No Y is Z No men are gods.
 Some X is Y Some living beings are men.

The premises are now in *Ferio*, whence,

Some X is not Z Some living beings are not gods.

The modes of the Fourth Figure, are thus, with the appearance of great inversion, mere varieties of the primary Figure. The transposition of the order of the premises is the most insignificant of all the alterations made on a syllogism. It signifies nothing to the reasoning, in what order the premises are stated. The three first moods depart from the standard moods in very little besides. The two last moods, as has

been seen, present both premises converted; and the first of the two is superfluous, even as a form.

The prime importance of the Syllogism attaches to its standard forms, that is, to the First Figure. In it we learn the essential structure of each valid deduction—a universal ground proposition, affirmative or negative, and an applying proposition, which must be affirmative. These appear, in the standard syllogism, in the order stated—first, the ground proposition (the major premise), secondly, the applying proposition (the minor premise). In the subsequent figures, these are sometimes transposed; and, in two forms, *Baroko* and *Bokardo*, they are greatly disguised. The ground proposition is called by Hamilton the *sumption*, the applying proposition, the *subsumption* (more strictly, the *subsuming* proposition).

It is not easy at first sight to point out any of the forms of the 2nd, 3rd, or 4th Figures that are of special importance in the conduct of reasoning or argumentation. The Fourth Figure is the least important of all; next, perhaps, the second, which, with the exception of *Baroko*, scarcely disguises the standard forms. The Third Figure is useful in overthrowing universal propositions, by exceptions or contradictory particulars.

It was pointed out by Aristotle, that in the First Figure only have we conclusions in all the forms, A, E, I, O. The Second Figure is restricted to negative conclusions; the Third Figure, to particulars. The Fourth Figure, which Aristotle did not recognize, does not admit of a universally affirmative conclusion.

In explanation of the possible uses of the Figures after the first, two circumstances may be remarked that lead to departures from the typical form. In the first place, the order of subject and predicate in either premise, and consequently the figure wherein the syllogism naturally falls, may vary with the idea uppermost in the mind of the reasoner. "The best form of Government is Government by a plurality of persons," and "Government by a plurality of persons is the best form of Government," are variations of the same statement that would cause a variation of figure. In the second place, the extent of the middle term relatively to the extent of the major and minor, gives rise to variations. When the middle term is larger than either major or minor, it naturally forms the predicate both of the major and of the minor premise, producing a syllogism of the Second Figure. When, again, the middle term is smaller than either, it naturally forms the subject of both premises, producing a syllogism of the Third Figure.

It has been shown in the detailed explanation above given, that the fifteen moods of the three last Figures are strict equivalents of the Moods of the First Figure, and therefore have the same validity as these standard moods. The demonstration of this equivalence is technically called the *Reduction* of the syllogisms, or their revocation to the primitive forms of affirmative and negative predication. The necessity of Reduction depends upon the nature of the proximate canons adopted for the syllogism. If those canons are applicable only to the First Figure, then, before we can test the validity of irregular moods, we must reduce them to moods of the First Figure. If the proximate canons are applicable directly to all syllogistic moods, reduction is unnecessary.

Order of the Premises. Many logicians have inverted the order of the premises, commencing with the minor. Thus—

All X is Y

All Y is Z

All X is Z.

This is the form that seems most convenient and convincing, in a chain of reasoning, as in the *Sorites*. It suits the particular form of the syllogistic axiom, expressed by 'the mark of a mark is a mark of the thing;' X is a mark of Y, Y is a mark of Z; hence X is a mark of Z. It, however, disguises the genuine type of Deductive Reasoning, which ought to be exhibited in the standard syllogism, even, if we depart from it in the other figures. The universal proposition is rightly put forward as the foundation of the reasoning; to which should follow the applying premise, or the minor. In the moods of the 2nd, 3rd, and 4th Figures, inversion of premises occurs as one form of departure from the First or regular figure.

Aristotle's mode of writing *Barbara* is—

A is predicated of all B

B is predicated of all C

A is predicated of all C—

where the minor is given first, and the propositions inverted in the wording; 'A is predicated of all B,' is the same as All B is A.

6. The Mnemonic Lines of the Syllogism contain the statement of the different moods, with the manner of reducing to the First Figure, those of the three last Figures.

To each of the moods, as described, a technical name has been appended, *Barbara*, *Celarent*, &c. These words have

been constructed for showing the constituent propositions of each mood, and how the moods of the 2nd, 3rd, and 4th Figures may be transmuted into moods of the 1st Figure; as in the process actually gone through in the foregoing explanation.

The names are made up in lines of Latin hexameter verse. Among artificial aids to memory, they stand unrivalled:—

Fig. 1. bArbArA, cElArEnt, dArII, fErIOque, prioris.

Fig. 2. cEsArE, cAmEstrEs, fEstInO, bArOkO, secundæ.

Fig. 3. tertia, dArAptI, dIsAmIs, dAtIsI, fElAptIn, bOkArDO, fErIsO, habet: quarta insuper addit.

Fig. 4. brAmAntIp, cAmEcEs, dImArIs, fEsApO, frEsIsOn.

Each of these names represents a mood; the three capital letters in each standing for the three propositions, as symbolized in their Quantity and Quality by the forms A, E, I, O. Of the smaller letters, or consonants, *r*, *n*, *t*, are meaningless or dumb letters. The consonants that commence each name—*b*, *c*, *d*, *f*—indicate the moods in the First Figure that the several moods in the other Figures are reduced to; *Bramantip* is reduced to *Barbara*, *Cesare* to *Celarent*, and so on. The consonants *m*, *s*, *p*, and *k*, signify the processes of Reduction: *m* indicating that the premises have to be transposed; *s* indicating simple conversion; *p* conversion by limitation, or *per accidens*; while *k* is the symbol of *reductio ad impossibile*. The application of each is to the vowel immediately preceding. Thus, in *Bramantip*:—

All Z is Y

All Y is X

Some X is Z—

we learn from *m* that to obtain the form of *Barbara*, the first mood of the First Figure, we must transpose the premises. And as we should then see ourselves entitled to conclude 'All Z is X,' it has further to be signified by *p*, that to obtain the conclusion 'Some X is Z,' we must make a limited conversion. So in *Fesapo* to obtain *Ferio* of the First Figure, we must convert E simply, and A by limitation. Although the method of reduction *ad impossibile* may be applied to any of the irregular moods, the letter *k* occurs only in two, *Baroko* and *Bokardo*, these being the only two that the logicians found irreducible by the processes of transposition and conversion.

7. The rules or Canons of valid reasoning are variously stated. They are proximate rules, being derived from the fundamental axioms of all Deduction.

Common Canons.—These are six in number.*

- (1) Every Syllogism has *Three*, and only three, *Terms*.
 - (2) There must be *Three*, and only three, *Propositions*.
 - (3) The *Middle Term* must be distributed once, at least, in the premises.
- That is to say, the Middle Term must be a universal in one or other of the premises. It must be the subject of a universal proposition (*All Y is Z, No Y is Z*), or else the predicate of a negative proposition (*No X is Y, Some X is not Y*). As the subject of a particular proposition (*Some Y is Z, Some Y is not-Z*), and as the predicate of an affirmative proposition (*All X is Y, Some X is Y*), the middle term Y is particular, or undistributed.

By a reference to the nineteen valid syllogisms, it will be seen that in each of them the middle term is distributed once in the premises. Thus, in the First Figure throughout, it is the subject of the major, which is a universal (*All Y is Z, No Y is Z*). This is as it ought to be in the standard syllogism. In the Second Figure, it is distributed twice in the major, and twice in the minor (*Some X is not-Y*). In the 1st, 2nd, 4th, and 5th moods of the Third Figure, it is distributed in the minor; being also distributed in the major, in the 1st and 4th. In the Fourth Figure, it is distributed in the minor, in all the moods but the last.

In the following couples, there is no distribution of the middle term (Y), and consequently none of the couples could stand as premises in a valid deduction.

| | | |
|------------|-------------|-------------|
| All Z is Y | Some Z is Y | All Z is Y |
| All X is Y | Some X is Y | Some X is Y |

| | | |
|--------------|-----------------|------------------|
| Some Y is Z, | Some Y is not Z | All Z is Y |
| All X is X, | All X is Y | Some Y is not X. |

A pretended syllogism, in such forms as these, or any form where the rule does not hold, is said to exemplify the fallacy of *undistributed middle*.

Such are the following :—

| | |
|-------------|--------------------------------|
| Some Y is Z | Some men are kings. |
| All X is Y | All cooking animals are men. |
| All X is Z | All cooking animals are kings. |

Other examples will occur afterwards.

(4) *No term undistributed in the premises must be distributed in the conclusion.* In other words, there must not be a greater

* After Whately, who gives them as a condensation of the twelve canons of Aldrich.

quantity attaching to any term in the conclusion, than is attached to the same term in the premises. If X be particular in the premises, so must it be in the conclusion; the same with Z. This condition, likewise, is fulfilled in the valid syllogisms. Thus :—

| | |
|------------|------------------|
| All Y is Z | No Y is Z. |
| All X is Y | Some X is Y. |
| All X is Z | Some X is not Z. |

In the first of the two, the subject of the conclusion, is universal in the minor premise, and may therefore be universal in the conclusion; in the second, it is particular in the minor, and must be particular in the conclusion. *In both*, the predicate of the conclusion is particular in the premises, and must be particular in the conclusion. So if, in *Darii*, a universal conclusion were drawn, it would be invalid.

| | |
|-------------|---------------------------------|
| All Y is Z | All men are mortal. |
| Some X is Y | Some extended things are men. |
| All X is Z | All extended things are mortal. |

We may have premises, free from the last-named vice of undistributed middle, yet made to yield a false conclusion by overstepping the present rule, or raising a term of particular quantity, in the premises, to the rank of universal quantity in the conclusion. To this error is given the name, *Illicit process*; and according as the unduly extended term occurs in the major or in the minor premise, the error is called illicit process of the *major* or illicit process of the *minor*.

In the foregoing instance, the illicit process is in the minor. We give an instance of illicit process of the major.

| | |
|-----------------|--------------------------|
| All Y is Z | All men are fallible |
| Some X is not Y | Some beings are not men. |
| No X is Z | No beings are fallible. |

The major term 'fallible,' being the predicate of an affirmative proposition, is particular or undistributed; in the conclusion, it is the predicate of a negative proposition, and is therefore distributed.

(5.) *There can be no conclusion drawn from negative premises.*

| | |
|-----------|------------------|
| No Y is Z | No men are gods |
| No X is Y | No trees are men |

do not supply the materials for a deductive inference. The reason of this is already apparent from what has been said as to the applying proposition, which must always *affirm*. To know only that two things are each excluded from a third thing is to know nothing concerning their mutual relation.

(6.) *If one premise be negative, the conclusion must be negative.*

This is illustrated throughout the series of valid syllogisms.

If one premise be negative, all that is predicated concerning one of the terms is its exclusion in whole or in part from the middle term: we cannot, therefore, conclude through the medium of the middle term anything about its total or partial co-extension with the other term.

In order to facilitate the detection of unsound syllogisms, the two following rules, directly deducible from these canons, are also enounced.

A. There is no inference from *particular premises*.

Some Y is Z Some Y is Z

Some X is Y Some X is not-Y

give no conclusion. The first example contains an undistributed middle; and the weakest inference drawn from the second (Some X is not Z) would contain an illicit process of the major.

B. If one premise is particular, *the conclusion must be particular*.

As in *Darii*, *Ferio*, &c.

Any attempt to extract a universal conclusion where both premises are not universal would incur either undistributed middle or illicit process.

This last canon, and also the Sixth, are embraced in one statement—'The conclusion always follows the weaker part.'

8. *Hamilton's Canons*. These are three in number. The first contains the 1st and 2nd of the foregoing list (Three Terms and Three Propositions). The two others are as follows:—

II. Of the Premises, the Sumption must in Quantity be *definite* (i.e. universal or singular); the Subsumption in Quality *affirmative*.

As Hamilton means by the Sumption the universal or ground proposition, and by the Subsumption, the applying or subsuming proposition, this is declaring the characters of the standard syllogism. It appears that, through all the mutations of syllogistic moods, there must always be one universal proposition (or else a definite singular), and one affirmative proposition. (The meaning of the alternative, a *singular* proposition will appear afterwards).

III. The conclusion must correspond in *quality* with the Sumption, and in *quantity* with the Subsumption.

Whatever be the *quality* of the *Universal* or ground proposition, that must be the quality of the conclusion; the one

being affirmative the other is affirmative; the one negative, the other is negative.

Again, the *quantity* of the Applying proposition is the true quantity of the conclusion; universal giving universal, and particular giving particular.

These two rules of Hamilton's are given as the equivalent, for Whately's four last. They have the advantage of placing, in a due prominence the fundamental structure of deductive reasoning, which is altogether invisible in the foregoing canons, but they are not readily applicable to the more distorted figures. Before using them, we must first discover which term contains the sumption, and which the subsumption; and for this, we must refer to the directions given respecting the irregular moods. In short, we must first redress the inversions and distortions of the irregular moods, which is substantially to go through the process of reducing each to the first figure.

9. *The rules of the syllogism given in the form of separate canons for each figure.* For the First or standard Figure, the canons of Hamilton are the most suitable expression. For each of the other Figures, special canons may be framed according to the nature of the Figure.

Thus, in the second Figure, it can be shown that,

(1) *One premise is negative.*

(2) *The major premise is universal.*

The proof is easy. (1) If both premises were affirmative, the middle term being the predicate of both premises, it would be undistributed.

Again, (2) if the major were particular, the weakest conclusion that could be drawn, Some X is not Z, involves illicit process of the major.

It follows from the first of the two rules (One premise must be negative) that, in this Figure, it is possible to prove negative conclusions only.

In the Third Figure, the canons are,

(1) *The minor premise is affirmative.*

(2) *The conclusion is particular.*

If the minor premise were negative, the conclusion must be negative, and the major term affirmative, which would involve an illicit process of the major.

Again, the conclusion must be particular, whether the syllogisms be affirmative or negative.

The minor premise being affirmative, there cannot be a uni-

versal affirmative conclusion without illicit minor. In a universal negative conclusion both terms are distributed: and they cannot both be distributed in the premises, unless both premises were negative, which could not be.

In the fourth Figure,

(1) *In the negative moods, the major is universal.*

| | |
|------------------|-------------|
| Some Z is not Y, | Some Z is Y |
| All Y is X, | No Y is X |

could not yield even particular conclusions, without illicit process of the major. We should have to infer—Some X is not Z: and Z is undistributed in the premises in consequence of the particularity of the major.

(2) *If the major is affirmative, the minor is universal.*

A particular minor to an affirmative major would give

| | |
|--------------|-----------------|
| All Z is Y, | All Z is Y |
| Some Y is X, | Some Y is not X |

both forms containing undistributed middle.

(3) *If the minor is negative, both premises are universal.* Try

| | |
|------------------|--------------|
| All Z is Y, | Some Z is Y, |
| Some Y is not X, | No Y is X. |

There is, in the first form, undistributed middle; and in the second, the weakest conclusion, Some X is not Z, contains illicit process of the major.

This rule is implied in the two preceding. By the First rule, the Major is universal, because the mood is negative. By the Second rule, the Minor is universal, because the major is affirmative.

(4) *If the minor is affirmative, the conclusion is particular.*

With minor affirmative, we have—

| | |
|-------------|-------------|
| All Z is Y, | No Z is Y |
| All Y is X, | All Y is X, |

In both cases, a universal conclusion would be attended with illicit process of the minor.

10. That the valid moods are those above given, and no more, is shown by testing all the other possible moods according to the syllogistic canons.

The possible moods may be arrived at by computing the possible groups of threes that can be made out of the four propositional forms—A, I, E, O. Now, taking the premises alone, there are sixteen different couples that can be made from these four letters.

| | | | |
|------|--------|------|--------|
| A, A | I, A | E, A | O, A |
| A, I | (I, I) | E, I | (O, I) |

| | | | |
|------|--------|--------|---------|
| A, E | I, E | (E, E) | (O, E) |
| A, O | (I, O) | (E, O) | (O, O). |

Of these sixteen forms, we can reject at once, as inadmissible, first, those that have both propositions particular—II, IO, OI, OO. We can farther reject those that have both negative—EE, EO, OE (OO is rejected on the previous ground). After these seven rejections, there are nine forms remaining.

For a farther sifting, two methods are open to us. First, let us try whether every one of the nine couples may stand as premises to conclusions of all the forms, A, I, E, O:

| | | | |
|-----------|-----------|-----------|-----------|
| A, A, A | (A, I, A) | (A, E, A) | (A, O, A) |
| A, A, I | A, I, I | (A, E, I) | (A, O, I) |
| (A, A, E) | (A, I, E) | A, E, E | (A, O, E) |
| (A, A, O) | (A, I, O) | A, E, O | A, O, O |

and so on through the remaining five forms.

Now, by applying the canon that requires a particular conclusion when one of the premises is particular, we exclude two in the second column—A I A, A I E, and two in the fourth—A O A, A O E. By applying the canon that requires a negative conclusion when one of the premises is negative, we exclude, in the third column, A E A, A E I; in the fourth column, A O I (also A O A excluded on the previous ground). Although no express canon is laid down requiring an affirmative conclusion from affirmative premises, such canon could be proved to be valid; and by means of it, two exclusions would be made in the first column—A A E, A A O, and one farther exclusion in the second. Hence, of the sixteen forms, six only survive these successive purgations. By a similar operation, extended to the remaining twenty forms, it would appear that there are in all twelve forms admissible;—

| | | | | | |
|--------|--------|--------|--------|--------|--------|
| A A A, | A A I, | A E E, | A E O, | A I I, | A O O |
| E A E, | E A O, | E I O, | I A I, | I E O, | O A O. |

If these twelve forms were each admissible in all the Figures, there would still be forty-eight valid syllogisms. But, by stating them under the successive figures, their ranks are thinned still farther. Thus, in the First Figure, A A I and A E O are superfluous because they infer a smaller conclusion when a larger could be drawn; with the premises A A, we can infer A (*Barbara*); with A E, we infer E (*Celarent*). Of the remaining ten, six would involve violations of fundamental canons, as may be seen by expressing them in full. Two examples are enough. Thus, A E E gives—

| | |
|------------|--------------------|
| All Y is Z | All men are mortal |
|------------|--------------------|

No X is Y

No molluscs are men

No X is Z

No molluscs are mortal

which contains illicit process of the major. The same would happen under a particular conclusion, as in A, E, O. Again, I, A, I—

Some Y is Z

Some fishes are sharks

All X is Y

All salmons are fishes



Some X is Z

Some salmon are sharks—

has the middle term undistributed.

By operating in this manner, we reduce the valid moods of the First Figure to the four formerly given—A A A, E A E, A I I, E I O.

The same process repeated for the remaining figures has the result of reducing the admissible forms to those actually given in the scheme of the syllogism.

The other method of elimination is to apply the special canons of the figures to the nine forms of unobjectionable premises, A A, A I, &c. By the canons of the standard syllogism, the major is universal and the minor affirmative; whence the forms, A E, A O, I A, O A, are rejected at once; and there remain only the four, A A, A I, E A, E I, corresponding to the four moods of the First Figure. For the Second Figure, the canons (One premise is negative; the major is universal) exclude A A, A I, I A, I E, O A; leaving A E (*Camestres*), A O (*Baroko*), E A (*Cesare*), E I (*Festino*). For the Third Figure, the first canon (The minor is affirmative) excludes A E, A O, I E; and there remain A A (*Darapti*), A I (*Datisi*), I A (*Disamis*), E A (*Felapton*), E I (*Ferison*), O A (*Bokardo*).

For the Fourth Figure, the first canon (In the negative moods, the major is universal) excludes I E, O A. The second canon (If the major is affirmative, the minor is universal) excludes A I, A O. The remainder are A A (*Bramantip*), A E (*Camenes*), I A (*Dimaris*), E A (*Fesapo*), E I (*Fresison*).

AXIOM OF THE SYLLOGISM.

11. Logicians have aimed at reducing the whole of the special canons or rules of the Syllogism to one comprehensive Law or Principle.

The oldest form of this principle is that named the *Dictum de omni et nullo*. 'Whatever is affirmed or denied of a class, is affirmed or denied of any part of that class.'

As stated, this maxim seems merely one of the forms of Immediate Inference:—'all men are mortal,' hence 'this man, ten men, some men, are mortal.' This, however, is not the

form actually assumed by the syllogism. We have to prove that some object is mortal, not expressly named a man, but designated by some other title, as 'king.' We cannot say 'men are mortal,' therefore 'kings are mortal;' such an inference can be made only through an *intermediate* assertion, 'kings are men.'

Another defect has been pointed out in the *dictum*; namely that it proceeds upon the old erroneous view of a proposition, the reference of a thing to a class. This, however, might be got over by understanding 'class' to mean the class *indefinite*, marked by the connotation of the class name. Practically, such must be the case; we have no means of pointing out the class 'men,' except as the possessors of human attributes.

Considering the *dictum* as the basis of all Deductive Reasoning, we might amend it thus:—'whatever is true of a whole class (class indefinite, fixed by connotation), is true of whatever thing can be affirmed to come under or belong to the class (as ascertained by connotation).' This supposes the need of a second affirmation, the minor proposition, and is no longer an immediate inference.

12. The defects of the *dictum* are supposed to be remedied by this form:—

Attributes, or Things, co-existing with the same Attributes or Things, co-exist with one another (Affirmative).

If the attributes of a king co-exist with those of a man, and the attributes of a man co-exist with the attribute 'fallibility,' the attributes of a king co-exist, or co-inhere with the attribute fallibility.

There is a close resemblance between the present form and the mathematical axiom—Things equal to the same thing, are equal. The two are alike axioms of *mediation*; they connect two things by a common third.

The negative form is stated thus:—'One thing co-existing with a second thing, with which second thing a third thing does not co-exist, is not co-existent with that third thing;' which resembles the axiom—Things unequal to the same thing, are unequal.

This mode of stating the axiom has often been adopted by logicians:—*Nota notæ est nota rei ipsius*; Things that agree with the same third, agree among themselves. For the negative form—*repugnans notæ, repugnat rei ipsi*; Things whereof the one agrees, the other does not agree, with the same third, do not agree among themselves.

The advantages of the form are indicated by the remarks already made. It gives very great prominence to the fact of *mediation* in Deductive Inference, and thus draws a broad line between it, and Immediate or Apparent Inference. It also accommodates itself to such a case as *Darapti*, with a singular subject, thus,

Socrates was wise.

Socrates was poor.

Some wise men have been poor.

Now, the treating of a *Singular* proposition as a universal, which is necessary to make the above a regular syllogistic form, has always seemed a great anomaly in the syllogism. Indeed, it is a subversion of the theory of Deductive Reasoning, as supposed to consist in the application of a general or universal principle to a case coming under it. But, if we accept the present form of the axiom, the above syllogism is rendered with apparent ease. 'Wise' co-exists with 'Socrates;' 'Poor' co-exists with Socrates; therefore 'Wise' and 'Poor' co-exist with one another; that is, 'Some wise persons are poor.'

A farther advantage of the same form consists in following out the 'Connotation' theory of Propositions. The extension of the several propositions is completely banished from it, and nothing but Connotation or Comprehension left. It is no longer 'all A is B,' but the attribute A co-exists with the attribute B, and so on. From the same cause, a seeming facility is given in chains of reasoning, which can be rendered thus:—A is a mark of B, B of C, C of D; wherefore A is a mark of D.

Notwithstanding so many advantages, this form of the axiom now described is unworkable as a basis of the syllogism. The fatal defect consists in this, that it is ill adapted to bring out the difference between *total* and *partial* coincidence of terms, the observation of which is the essential precaution in syllogizing correctly. If all terms were *co-extensive*, the axiom would flow on admirably; A carries B, all B and none but B; B carries C in the same manner; whence A carries C, without limitation or reserve. But, in point of fact, we know that while A carries B, other things carry B also, whence a process of limitation is required, in transferring A to C through B:—A (in common with other things) carries B; B (in common with other things) carries C; whence A (in common with other things) carries C. The axiom provides no means of making this limitation; if we were to follow A literally, we should be led to suppose A and C co-extensive: for such is the only obvious

meaning of 'the attribute A coincides with the attribute C.'

Unless the predicate is quantified, as Hamilton recommends, the propositional form in Extension—'all men are mortal,' does not explicitly suggest that 'men are but a part of mortals:' yet we can readily conceive the fact when reminded of it; the extent of 'mortal beings' is greater than the extent of 'men.' But the proposition stated in pure connotation or comprehension, as the present axiom requires,—'the attributes of men co-exist with the attribute mortality'—is difficult to adapt to the fact that mortals are more numerous than man. We should have to make a still greater circumlocution:—the attributes of men co-exist, but are not the only attributes that co-exist, with the attribute 'mortality.' So, the attributes of a king co-exist, but are not the only attributes that co-exist, with the attributes of men. The conclusion would then be—The attributes of a king co-exist, but are not the only attributes that co-exist, with the attribute 'mortality.' Now, as the axiom 'attributes co-existing with the same attribute co-exist with one another' does not suggest these necessary limitations, it is not, as worded, an explicit basis for the syllogism.

It is only the same objection, otherwise put, that the axiom does not accommodate itself to the type of Deductive Reasoning, as contrasted with Induction—the application of a general principle to a special case. Anything that fails to make prominent this circumstance is not adapted as a foundation for the syllogism.

The scientific processes of Induction and Deduction are habitually conceived on the basis of Extension; it is only thus that we readily appreciate the greater or less generality of propositions. Hence the proper view of the syllogism, as of the notion and the proposition, is to base it on Extension, but to determine the extension by Connotation or Comprehension. 'All men are mortal' is best understood as the concrete population of human beings, defined and determined by the class attributes of humanity. This double point of view complies with all the exigencies of reasoning, and is not advantageously surrendered in favour of the statement of propositions in pure comprehension.

The result of the comparison of the two axiomatic statements is, that the *Dictum de omni et nullo*, properly guarded, is the most suitable and exact representation of the essential feature of Deductive Reasoning or Syllogism.

The case of Singular Propositions, held for the nonce to be

universal, is a grave exception to the Deductive process as we have uniformly described it. On examining such cases, however, we may see good reason for banishing them from the syllogism. Let us take the example already quoted :—

Socrates is poor

Socrates is wise

Some poor men are wise.

- Properly, the conclusion is, 'one poor man is wise.' Now, if 'wise,' 'poor,' and 'a man,' are attributes belonging to the meaning of the word Socrates; there is then no march of reasoning at all. We have given, in Socrates, *inter alia*, the facts 'wise,' 'poor,' and 'a man,' and we merely repeat the concurrence, which is selected from the whole aggregate of properties making up the whole, 'Socrates.' The case is one under the head 'Greater and Less Connotation,' in Equivalent Propositional Forms, or Immediate Inference.

But the example in this form does not do justice to the syllogism of singulars. We must suppose both propositions to be real, the predicates being in no way involved in the subject. Thus :—

Socrates was the master of Plato

Socrates fought at Delium

The master of Plato fought at Delium.

It may fairly be doubted whether the transitions, in this instance, are anything more than equivalent forms. For the proposition, 'Socrates was the master of Plato, and fought at Delium,' compounded out of the two premises, is obviously nothing more than a grammatical abbreviation. No one can say that there is here any change of meaning, or anything beyond a verbal modification of the original form. The next step is, 'the master of Plato fought at Delium,' which is the previous statement cut down by the omission of 'Socrates.' It contents itself with reproducing a part of the meaning, or saying less than had been previously said. The full equivalent of the affirmation is 'the master of Plato fought at Delium, and the master of Plato was Socrates;' the new form omits the last piece of information, and gives only the first. Now, we never consider that we have made a real inference, a step in advance, when we repeat *less* than we are entitled to say, or drop from a complex statement some portion not desired at the moment. Such an operation keeps strictly within the domain of Equivalence or Immediate Inference. In no way, therefore, can a syllogism with two singular premises be viewed as a genuine syllogistic or deductive inference.

13. The Proof of the Axiom is uncontradicted experience.

The *Dictum* is not a mere rule of consistency, exacting the admission, in equivalent forms, of all that has been conceded in one form. It is a mediate process, and the mediation has to be justified by an appeal to the facts. As far as proof goes,

it resembles in character the second form above given—‘Things co-existing with the same thing, co-exist,’ and the mathematical axiom ‘Things equal to the same thing are equal.’ All the three principles stand upon the same foundation; some philosophers refer them to intuition, others to experience; but the mode of proof for one is the mode for all. The *dictum* seems to approach nearest to a mere rule of consistency; yet the fact of mediation makes all the difference; ‘the identical of an identical is identical’ is a new step and needs a new justification. Nobody would accept even so obvious an inference—as ‘men are mortal, kings are men. kings are mortal,’ without first verifying upon examples the peculiar kind of transition involved. We are so alive to the snares lurking in the most obvious and plausible forms of language, that we do not trust any of them without the check of actual trials. Nothing could seem more satisfactory than ‘A co-exists with B, B with C, therefore A co-exists with C wholly and unconditionally,’ yet until we have elaborately fenced the operation against the simple conversion of a universal, the conclusion is unwarranted.

Viewing together the Mathematical axiom of Equality and the axiom of the Syllogism, Mr. de Morgan remarks:—‘In both there is a law of thought appealed to on primary subjective testimony of consciousness;’ ‘equal of equal is equal’ in the one; ‘identical of identical is identical’ in the other. The two laws are equally necessary, equally self-evident, equally incapable of being resolved into simpler elements.

14. There are other modes of stating the Axiom. Hamilton has two forms. The first is for what he calls Informal Reasoning:—In so far as two notions (notions proper or individuals) either both agree, or one agreeing the other does not, with a common third notion; in so far, these notions do or do not agree with one another.

This is simply one way of wording the *Nota notæ*, and is liable to the objections urged against that form. There is no provision for distinguishing total from partial agreement, and therefore no basis for the working of the syllogism. The words ‘agreement’ and ‘disagreement’ are less apt than ‘co-existence’ and ‘non-coexistence’ for expressing the axiom, they have the defects inherent in the ‘judgment’ theory of Propositions.

15. For the Figured Syllogism, where the terms are related as subject and predicate of propositions in a given

order, Hamilton enounces this form:—What worse relation of subject and predicate subsists between either of two terms and a common third term, with which one, at least, is positively related; that relation subsists between the terms themselves.

'The peculiar phraseology 'What worse relation' is a manner of saying that the conclusion must carry the weakest relationship signified by the premises. If there be a negative in the premises, there must be a negative in the conclusion; if there be particularity in the premises, there must be particularity in the conclusion. The same thing is otherwise expressed—'The conclusion must follow the weaker part.'

This is the Axiom given in Extension, and is in accordance with the *Dictum*, although not stated with the same generality. It more resembles one of the canons for working out the syllogistic details, itself resting on the *Dictum*.

16. The first of Hamilton's two forms is expressed otherwise thus (Thomson):—The agreement or disagreement of one conception with another, is ascertained by a third conception, inasmuch as this, wholly or by the same part, agrees with both, or with only one of the conceptions to be compared.

This form appears to be based upon Comprehension, or the *Nota note*, but endeavours to introduce the limitations requisite for discriminating total and partial quantity. The phraseology, however,—'conception, &c.'—is ambiguous; it may express either extension or comprehension—'men' or the attributes 'human.' If, taken in extension (which is most probable), it closely reproduces Hamilton's second form, and puts stress upon the difference between total and partial coincidence. Nevertheless, it does not rise to the sweep of the *Dictum*, in declaring the paramount circumstance of deductive reasoning,—the carrying out of a general law to particular cases.

If 'conception' means attributes, comprehension, or connotation, the phraseology would indicate Hamilton's syllogism of Comprehension, and would not suggest the common syllogism. The attributes 'king' and the attribute 'mortal' agree (better 'coincide') by agreeing (coinciding) with the same part of the attributes 'human.' Hamilton's syllogism is more explicit; thus—The attributes 'king' contain the attributes 'man'; the attributes 'man' contain the attribute 'mortal'; the attributes 'king' contain the attribute 'mortal.'

17. In the comprehensive scheme of De Morgan, the axiom is a generalization of many special axioms. The syllogism is treated as the composition of two relations into one; the axiom is 'the relation of a relation is a relation compounded of the two.'

The truth of this is seen, and its application controlled, by the special instances of relationship. One of these instances is the axiom of the common syllogism. Others are the mathematical axioms, 'Equal of equal is equal,' and 'greater of greater is still greater' (*a fortiori*). Among more special instances are 'antecedent and consequent,' 'ancestor and descendant.'

18. It has been supposed by some that the common axiom, as expressed by the 'dictum de omni et nullo,' is a consequence of the Laws of Thought (Identity, Contradiction and Excluded Middle).

Hamilton maintains that categorical syllogisms are regulated by the fundamental laws of Identity and Contradiction. He interprets the law of Identity as the identity of a whole and the sum of its parts, whence he considers it right to infer that what belongs to a whole belongs to its part. Mr. Mansel agrees with Hamilton in referring the syllogistic laws to the same principles.

The effect of this doctrine is to abolish the difference between Immediate and Mediate Inference, by bringing mediate inference under Immediate, or under the law of Consistency. On the face of it, the supposition is unlikely; and accordingly it has been denied by other logicians. Thus, Mr. de Morgan (*Syllabus*, p. 47) remarks of the attempts to reduce the syllogism to the three so-called Laws of Thought, 'When any one attempts to show *how*, I shall be able to judge of the process; as it is, I find that others do not go beyond the simple assertion, and that I myself can detect the *petitio principii* in every one of my own attempts.'

The law of Consistency requires us to concede that what is true of a class is true of every individual in the class; 'all men are fallible,' 'the half of men are fallible, this man is fallible'; here there is no transition, it is the same fact, repeated only to a less extent. But when we say 'kings are men,' 'kings are fallible,' there is a transition to a different subject, a subject not present to the mind as a part of the original whole, but brought under it by a second assertion. Now a distinct axiom,

is needed to transfer the attribute under this new case. The axiom may be in its nature self-evident, but the conclusions regulated by it are not identical with either of the premises, as an immediate inference, properly so called, is identical with the original form.

19. The special canons of the Syllogism are derivable from the Axiom.

(1) It easily follows from the *Dictum*, as explained, that there are three terms, and no more. There is a Universal Proposition containing a subject and a predicate, an applying or Interpreting proposition, adding a third term, and repeating one of the terms of the universal:—All or no Y is Z, All X is Y. The conclusion contains no new term—All X is Z. Whence there are three terms in all.

(2) The same examination shows that there are three and no more than three propositions;—the Universal, the Interpreting Proposition, and the Conclusion.

(3) The third special canon is—‘The middle term must be distributed once in the premises.’ Distribution or Universal Quantity in the middle term is essential to the total coincidence or non-coincidence of, with at least one of the other terms (*the middle term*); without which the two extreme terms could not be shown either to coincide, or not to coincide, in whole or in part. ‘Some men are fallible,’ ‘kings are some men,’—would not bring about a coincidence between ‘fallibility’ and ‘kings;’ one portion of men might be fallible, and a different portion might be kings. This is obviated if fallibility adheres to *all* men; it must then adhere to whatever objects are found to be men.

(4) The fourth special canon is—‘No term undistributed in the premises must be distributed in the conclusion.’ It may be brought under the *Dictum* thus:—The distribution of a term in the conclusion means universal or total coincidence with the other term of the conclusion;—‘All X is Z’ means that X is wholly coincident with, wholly included in Z. Now X and Z are brought together by a middle term Y; and if X did not wholly coincide with Y in the first instance, it could not be transferred, in total coincidence, to Z. If we had only some X is Y, even although all Y is Z, we could not declare *all* X to be Z. There is carried over to Z only so much of X as goes with Y; if that be the whole, the whole is carried; if a part, part is carried. If ‘all men are fallible,’ and ‘some beings are men,’ only *some* beings are fallible, namely, as many as are men.

(5) 'From negative premises, there is no inference.' Negative premises do not comply with the essential fact of the interpreting proposition, which is to declare that a given case comes under the sweep of the rule. Whether the universal be affirmative or negative, the applying proposition must, from its nature, be affirmative. No \bar{Y} is Z , no X is Y , could not be the means of bringing X under Z , or of bringing these two terms together in a conclusion; we could not, from such premises, infer even No X is Z . 'No matter is destructible' requires to be followed up with 'ether is matter' to prove that 'no ether is destructible.'

(6) 'If one premise be negative, the conclusion is negative,' expresses exactly what happens in the negative form of the axiom.

In the enlarged scheme of De Morgan, some of these rules are violated in appearance, but only in appearance. Thus from 'two negative premises' he draws a conclusion in the affirmative. This, however, arises from the elasticity of expression allowed by the use of contrary forms. Every affirmative proposition may be given as a negative; and there may be the semblance of negation, with the reality of affirmation in conformity with the axiom. Thus—

All Y is Z = No Y is not Z .

All X is Y = No X is not Y .

All X is Z = All X is Z .

20. The axioms—'Equals added to equals, give equal sums,' and the *argumentum a fortiori*, if received as axioms in Logic, are distinct from the axiom of the Syllogism, and must be independently proved.

The *argumentum a fortiori* is represented thus:—If A is greater than B , and B greater than C , still greater is A than C . This, and the other axiom stated, are purely mathematical in their character; they serve for the comparing of quantities as equal or unequal. They rest on their own special evidence of fact.

It will be seen that Boole draws the Syllogism under the axiom that suffices for the reduction of equations. He assumes that the analogy of the logical method and the algebraical is sufficiently close to allow of the substitution.

The conflicting opinions as to the evidence of axioms generally, whether of logic, of mathematics, or of other sciences, will be discussed in a succeeding chapter. [Regarding the Axiom of the Syllogism in particular, see the Supplementary Note at the end of the volume.]

EXAMPLES OF THE SYLLOGISM.

21. The chief application of the theory and the forms of the syllogism is to detect fallacies in deductive reasonings.

There are certain forms of deductive reasoning or argument, that are specious to appearance, and fallacious in reality; and the analysis of the syllogism is useful in disclosing the fallaciousness.

22. The course of procedure, in dealing with an argument in any way uncertain or perplexed, is as follows:—

I. Ascertain what is the conclusion, or the point to be proved. State this distinctly in a proposition so as to distinguish the Subject (*minor* term of the syllogism) and the Predicate (*major* term).

II. Find out the *middle* term of the argument. In a valid syllogism there must be a middle term, and only one: and it must be something not occurring in the conclusion.

III. Find out some proposition connecting the middle term with the major term; this is the *major premise* of the syllogism. Also some proposition connecting the middle term with the minor term; giving the *minor premise* of the syllogism.

IV. The two premises and the conclusion being stated in form and order, the validity may be judged according to the laws of the syllogism.

(1) If the deduction coincides with any of the valid moods, it is valid; if not, not.

(2) It being seen what Figure the argument comes under, it may be tested by the special canons of that figure.

(3) The general canons of the syllogism may be applied to discover errors, if there be any such.

Any one of these three modes may be adopted at choice; inasmuch as each of them singly is conclusive.

The easiest remembered mode of testing a syllogism, when once in form, is by the six general canons of the syllogism. Of these, the two that are most usually violated in sophistical reasonings are the 3rd (Distribution of the Middle Term) and the 4th (The quantity of the terms in the conclusion not greater than in the Premises). An argument with negative premises (5) would deceive no one. It would also be obvious, without much Logic, that one premise being negative, the conclusion must be negative (6).

23. As an alternative, we may discard the consideration

of the separate Figures, and reduce every argument at once to the standard form of Deduction.

From the very nature of deductive reasoning, the conclusion is a special application of some more general proposition. This more general proposition must be found in the premises ; it is the ground proposition ; in Hamilton's phraseology, the *Sumption*. There must also be found another proposition declaring its applicability to a particular case, namely, the case given in the conclusion. These two indispensable propositions may occur under distorted forms, which we must be able to redress by the methods already pointed out, that is, by obversion and conversion, as the case may be. Also, the conclusion may require to be obverted or converted, or both. By such methods, we may evade all the variations of figure, and come at once to the regular type of deduction.

EXAMPLES.

| | | |
|--------------------|-----------------|------------|
| All men are mortal | All Y is Z.—(A) | } 1st Fig. |
| No dogs are men | No X is Y.—(E) | |
| No dogs are mortal | No X is Z.—(E) | |

(1) This syllogism is in the First Figure, but there is no mood in that Figure containing the propositions A, E, E.

(2) Otherwise : The major term, mortal, is distributed in the conclusion, and not in the premises ; there is illicit process of the major.

(3) Or lastly : It contradicts the canon of the normal syllogism, whereby the minor is declared to be affirmative.

| | | |
|-----------------------|---------------|------------|
| All planets are round | All Z is Y.—A | } 2nd Fig. |
| A wheel is round | All X is Y.—A | |
| A wheel is a planet | All X is Z.—A | |

(1) There is no such mood in the Second Figure.

(2) The middle term, 'round,' is undistributed.

(3) There is a violation of the special canon of the Second Figure—One premise must be negative.

'Every honest man attends to his business ; this person attends to his business ; this person is an honest man.' This is the exact counterpart of the foregoing. The conclusion being 'this person is an honest man ;' the *minor* term is 'this person,' the *major*, 'an honest man.' The *middle* term is 'attends to his business.' The major premise (major and middle), 'Every honest man attends to his business,' A ; the minor premise, 'this man attends to his business,' A (a definite

individual may be considered as either A or I). On any one of the three grounds given in the foregoing example, the reasoning is fallacious.

These two examples are regarded by logicians as of a type calculated to mislead, and therefore exemplifying the use of the laws of the syllogism. It is interesting to enquire what circumstance gives them their fallacious plausibility. With this view, we may proceed by the alternative method above pointed out, namely, by ascertaining whether these be the regular premises of deduction.

To prove that a wheel is a planet, we must have a more general proposition, of which this shall be a particular case. Such a proposition would be 'all round bodies are planets.' We should then require an applying or subsuming proposition, namely, 'wheels are round bodies.' With these two propositions, the conclusion would be legitimate, that wheels are planets. Looking at the premises given, however, we do not find a proposition corresponding to the first, or the general proposition. It is stated, not that 'all round bodies are planets,' but only that 'all planets are round,' a different proposition. The confounding of the two is effected by the *simple conversion of a universal affirmative*; by arguing from 'all planets are round,' that 'all round bodies are planets,' which we can do only if there are no round things but planets. In short, the fallacy, traced to its root, is a *fallacy of conversion*; and if we are liable to be deceived by such syllogisms as the present, it is because we are liable to slip into this fallacy. There is something in the form of the universal affirmative that throws us off our guard; from the expression All X is Y, we are apt to assume the *co-extension* of X and Y, unless cautioned and educated to the contrary. In cases where the co-extension exists, and only in such cases, could the argument in question give a sound conclusion. Thus—

All matter gravitates.

Air gravitates.

Air is matter.

Now, by the same process as before, it is shown that the general proposition needed for this conclusion is 'All gravitating things are matter,' which happens to be true, but is not justified by the assertion in the major, 'all matter gravitates;' for there might be other gravitating things.

So in the second example "Every honest man attends to his business," &c., we should require the terms 'honest man' and 'attention to business' to be co-extensive, which they are not.

Whatever tendency we have to be deceived by such reasonings depends solely upon the intellectual weakness of presuming co-extension of terms, in universal affirmations.

Hume says:—‘ We have no perfect idea of anything but a perception. A substance is entirely different from a perception. We have therefore no idea of substance.’

The first step is to resolve the conclusion into its two terms. As often happens, in Logic, these terms are not the grammatical subject and grammatical predicate; a transformation must be given to suit the tenor of the premises. Comparing the first proposition with the last, we see that the *minor* term, or subject of the conclusion, must be ‘having an idea,’ the *major* term is ‘substance.’ The affirmation is negative; literally, our ‘having an idea’ is not true of substance. It is denied that substance is one of the things included under having an idea. The next point is to single out the *middle* term, namely, ‘perception.’ Joined with the major and minor terms respectively, this yields as premises—

No ‘having an idea’ is not perception.

All substance is not perception.

No ‘having an idea’ is true of substance.

In the present form, the reasoning is wholly inadmissible; the premises are both negative. We might, however, obvert the middle term ‘perception,’ and regard not-perception as the true middle (like changing ‘not wise’ into not-wise, or foolish). We have thus—

| | | |
|---------------------------------------|---|-------------------------------|
| No ‘having an idea’ is not-perception | E | } 2nd Fig. (<i>Cesare</i>). |
| All substance is not-perception | A | |
| No ‘having an idea’ is substance. | E | |

In this form the argument is sound.

It is often desirable to express arguments of great subtlety, such as the present, in the standard form of deduction. The requisite transmutation would have to be effected thus. The conclusion, ‘“having an idea” is not true of substance,’ is to be converted ‘No substance is included in our having an idea.’ For this, the universal proposition would be a proposition of denial more comprehensive than substance:—No not-perception is included in our having an idea. The minor is then, All substance is not-perception; whence we conclude according to the regular form for the negative deduction. From the middle term being a negation, however, this, can never be an easy form of argument; and more especially so in

the present argument, where perception is as wide as existence, and has only a formal, and not a real obverse.

Thus, then, in the First Figure, as *Oelarent*—
 Nothing that is not a perception (no not-perception) can
 be perfectly conceived, E.
 Substance is not a perception (a not-perception), A.
 Substance cannot be perfectly conceived. E.

• None but Whites are civilized; the Hindoos are not Whites; therefore they are not civilized.'

In a syllogism thus:—

No not-Whites are civilized E }
 The Hindoos are not Whites A } (*Oelarent*).
 The Hindoos are not civilized E }

A correct argument, the middle term being 'not-Whites,' for which the positive equivalent would be the remaining members of the Universe, 'races of men' (Black, brown, yellow, &c.) This would give a more intelligible form:—

No communities of the black, brown, or yellow races are civilized;

The Hindoos are of the black or brown races.

The Hindoos are not civilized.

• Abstinence from the eating of blood had reference to the divine institution of sacrifices; one of the precepts delivered to Noah was abstinence from the eating of blood; therefore, one of the precepts delivered to Noah contained the divine institution of sacrifices' (Whately).

Although prolix in the wording, there is little distortion in this example. The *minor* term is obviously 'one of the precepts delivered to Noah,' the *major*, 'contained or had reference to the divine institution of sacrifices.' The middle term is 'abstinence from the eating of blood;' and the arrangement is exactly as in the standard syllogism.

'Few treatises of science convey important truths, without any intermixture of error, in a perspicuous and interesting form; and therefore, though a treatise would deserve much attention which should possess such excellence, it is plain that few treatises of science deserve much attention.' (Whately).

The conclusion: gives as *minor* term 'few treatises of science,' as *major* 'deserve much attention' The *middle* term is 'convey important truths, &c.' The major premise, therefore, is—

All treatises of science that convey &c., deserve attention:
The minor premise—

Few treatises of science are works conveying important, &c.
The conclusion—

Few treatises of science deserve attention (*Darii*).

It was formerly remarked (p. 82) that for Some, in the minor term, we may have—Few, most, many, one, two,—provided that the same quantity is used in the premises and in the conclusion.

‘Enoch (according to the testimony of Scripture) pleased God ; but without faith it is impossible to please Him ; therefore Enoch had faith’ (Whately).

The minor and major terms are obvious. The middle is ‘pleasing God.’ The major premise is—‘pleasing God is impossible without faith,’ which is a circumlocution by way of expressing emphatically the proposition ‘pleasing God is having faith’—‘all persons that please God have faith.’ The minor premise being ‘Enoch pleased God,’ the conclusion follows from the regular type of deduction.

It was said by some one during the Reform discussions of 1867 :—‘Every reasonable man wishes the Reform Bill to pass. I don’t.’ There was but one inference. The speaker was not a reasonable man (*Camestres*). This is a good example to show that an effective argument may be given out of the First Figure.

If we follow the ordinary method of reduction in this case, we find ourselves in a difficulty. *Camestres* is usually reduced to the First Figure by transposing the premises and simply converting the original minor: if we do so in this case, we find a singular proposition in the major premise, which cannot be converted without doing great violence to the ordinary forms of language, and cannot stand as the grounding proposition conceived as a general rule. The general rule in this case is obviously the existing major—‘Every reasonable man wishes the Reform Bill to pass.’ But if we view this as the general rule, then we appear to have a *negative* applying proposition—‘I don’t.’ Looking more closely at the premises, we see that the true nature of the predication is disguised. The major proposition is really negative, and the minor really affirmative. The remedy for the distortion is to obvert the major into—‘No reasonable man wishes the Reform Bill to fail ;’ or ‘No man that wishes the Reform Bill to fail is reasonable.’

The minor when altered to correspond becomes—‘I do;’ and we have a syllogism in *Celarent*.

Another example of this same mood, *Camestres*, illustrates the occurrence in ordinary reasoning of other syllogistic forms than the moods of the standard figure. We are presented with the assertion that ‘No despotism is a good form of government,’ and on asking the ground of such an assertion, are told—‘Every good form of government promotes the intelligence of its subjects, and no despotism does that.’ This is an argument in *Camestres*.

| | |
|---|--------|
| Every good form of government promotes the intelligence of its subjects. | } cAm |
| No despotism promotes, &c. | |
| No despotism is a good form of govern- ment. | } trEs |

The above statement of the Major is the natural statement of the proposition; the order of subject and predicate is such as a reasoner would naturally observe. That it promotes the intelligence of its subjects is affirmed of every good form of government; the order of the terms conforms to the usual arrangement of having the largest term in the predicate; other agencies than good government promote the intelligence of the people.

As in the former *Camestres*, this syllogism cannot be reduced to the First Figure by the process indicated in the Mnemonic letters without putting the real Major, or grounding proposition, in the Minor place. We may retain the present order without violating the rule that the applying proposition must be affirmative. For the present major, affirmative in form, is obviously negative in its bearing; while the minor, negative in form, is really of an affirmative nature, asserting that a despotic form of government possesses the character contemplated in the ground proposition as precluding the title of good. By obverting the predicate of the major, the middle term, we manifest the real character of the premises:—

No form of government that fails to promote the intelligence of its subjects is a good form of government.

A despotism fails to promote the intelligence of its subjects.

No despotism is a good form of government.

In speaking of the general uses of the Figures, we remarked that the Third Figure is sometimes useful in making good an unobtrusive and timid contradictory. The three first moods

supply mild contraries to a universal negative; the three last mild contraries to a universal affirmative. We give an example of each.

Suppose a speaker to maintain absolutely and without reservation that speculation is of no value. His position in logical form is—‘No speculation is valuable.’ We subvert this and extort from the speaker a concession that his position is too extreme, when we obtain his assent to the two propositions—‘Some truths affecting human conduct are speculations’, and ‘All truths affecting human conduct are valuable.’ These two propositions involve the sub-contrary of the extreme negative;—namely, *Some speculations are valuable*. They are given in the order of subject and predicate natural to the occasion, and they fall into the Third Figure. They serve as premises either for *Disamis*, or *Datisi*, according to the order we observe in enouncing them. Thus:—

| | | |
|-------------------------------------|---|-----|
| Some truths affecting human conduct | } | dIs |
| are speculations | | |
| All truths affecting human conduct | } | Am |
| are valuable | | |
| Some speculations are valuable | | Is |

This is a syllogism in *Disamis*. But it is to be observed that we invert the normal order of the major and minor terms in the conclusion. The most natural form is *Datisi*—thus:—

| | | |
|-------------------------------------|---|-----|
| All truths affecting human conduct | } | dAt |
| are valuable | | |
| Some truths affecting human conduct | } | Is |
| are speculations | | |
| Some speculations are valuable | | I |

If our opponent should concede that *all truths affecting human conduct are speculations*, we should have a syllogism in *Darapti*. In that case, our partial contradiction would seem peculiarly bland, because our premises would then be superfluously strong, and we should have the appearance of remitting something in the conclusion.

Our next example illustrates the partial subversion of a universal affirmative by making good its sub-contrary, a particular negative. It is maintained that no attention should be given to what is not practical. This may assume the logical form of a universal affirmation,—‘Everything that is unpractical should be neglected.’ Desiring to Contradict this in a mild form, we may use the following argument:—

| | |
|--|-------|
| No truth applicable to practice should be neglected. | } fEl |
| Every truth applicable to practice may seem unpractical. | |
| Some seemingly unpractical truths should not be neglected. | } tOn |
| | |

This is a syllogism in *Felapton*. The major—'Some truths applicable to practice should not be neglected,' would equally suit our purpose, and with the above minor would give a *Bokardo*. In such cases as the above, it is difficult to say which is the grounding proposition. There is no violation of the essential nature of Deduction in regarding a particular proposition, or approximate generalization, as the ground of the argument. To make the reasoning a genuine deduction, it is required only that the grounding proposition be more general than the conclusion.

Arnauld's Universal Test.

It may be worth while to give an example of Arnauld's mode of testing a deductive argument without reference to its logical form.

He directs the pupil simply to observe whether the conclusion is contained in the premises. He gives the following example of his method :—

'I am in doubt whether this reasoning be good :—

The duty of a Christian is not to praise those that commit criminal actions.

Now those that engage in a duel commit a criminal action.

Therefore it is the duty of a Christian not to praise those that engage in duels.

'Now I need not trouble myself as to the figure or mood to which this may be reduced. It is sufficient for me to consider whether the conclusion be contained in one of the two first propositions, and if the other show this. And I find at once that the first proposition, since it differs in nothing from the conclusion, except that there is in the one, *those that commit criminal actions*, and in the other *those that engage in duels*,—that in which there is *commit criminal actions*, will contain that in which there is *engage in duels*, provided that *committing criminal actions* contains *engaging in duels*.

'Now it is clear by the sense that the term *those that commit criminal actions* is taken universally, and that it extends to all those that commit any such actions whatever ; and thus the

minor, *Those that engage in a duel commit a criminal action*, showing that *to engage in a duel* is contained under this term, *commit criminal actions*, shows also that the first proposition contains the conclusion.

This test of Arnauld's is the simplest of application to premises not couched in syllogistic terms. It is easily applied in any case: the only change of form that could aid in the scrutiny, would be to make the containing proposition of the same form with the conclusion.

To the following arguments, the student may supply such grounding propositions as would give them validity:—

A true philosopher is independent of the caprices of fortune, for he places his chief happiness in moral and intellectual excellence.

A slave is a human being, therefore he should not be held in bondage.

Not being thirsty, he cannot be suffering from fever.

The Reformation was accompanied and followed by many disturbances, and is therefore to be condemned.

Solon must be considered a wise legislator, seeing that he adapted his laws to the temper of the Athenians.

He was too impulsive a man not to have committed many errors.

Educated among savages, he could not be expected to know the customs of polite society.

Not every advice is prudent, for many advices are not safe.

Many assertions that are open to doubt are nevertheless worthy of attention, for many assertions that are open to doubt may be true.

'Napoleon never cared for anybody but himself.' In modified opposition to this, it may be urged that, after all, 'he was human.' Supposing this rejoinder is intended to establish that Napoleon *had some* disinterested affections, what grounding proposition does it require?

In like manner, subvert the assertion, 'Napoleon never knew fear.'

Volcanic eruptions, earthquakes, and plagues cannot be interpreted as a warning to evil-doers, for they involve alike the innocent and the guilty.

Some dogs are useful animals, for is not the retriever useful?

All zeal is not virtuous, there being a zeal that has no discretion.

'Table-turning,' (you may say,) 'is a thing I don't under-

stand.' Admitting this, I ask you to construct in an affirmative form, an argument which would entitle you, logically, yet not convincingly, to deny the existence, of table-turning. (Spalding).

Miscellaneous Syllogisms.

'Suppose a man says, 'I dislike all foreigners;' find a premise which, with his own assertion, would entitle him to say also, 'No foreigner deserves to be liked.' (Spalding).

All cold is to be expelled by heat: this person's disorder is a cold; and must therefore be expelled by heat.

No carnivorous animals have four stomachs: all ruminants have four stomachs: no ruminants are carnivorous.

Some men of inferior ability are legislators. All peers are legislators, and some peers are men of inferior ability.

'No war is long popular: for every war increases taxation; and the popularity of anything that touches our pockets is very short-lived.' (Spalding).

He that will not learn cannot become learned. This being so, there are many clever young men that we cannot expect to become learned.

There is some anger that is not blameworthy. What premise do you need for the conclusion,—'Some passions are not blameworthy.'

'No truth is without result; yet many truths are misunderstood.' What is the conclusion?

Some deserve to be imitated that are nevertheless fools. Whoever speaks the truth deserves to be imitated.

Humanity is a moral virtue: the study of polite letters is humanity; the study of polite letters is a moral virtue.

White is a good fellow: if, therefore, linen is white, it is a good fellow.

'He that says you are an animal speaks truly: he that says you are a goose, says you are an animal; he that says you are a goose speaks truly.' (Arnauld).

'You are not what I am. I am a man: therefore you are not a man.' (Arnauld).

One symptom of the plague is fever; this man has fever; therefore he has the plague.

Some objects of great beauty answer no other perceptible purpose, but to gratify the sight: many flowers have great beauty; and many of them accordingly answer no other purpose but to gratify the sight.

Every good statesman is favourable to progress. Some

members of Parliament, not being favourable to progress, are not good statesmen.

'Unpleasant things are not always injurious; afflictions are often salutary.' Supply the missing premise.

John is taller than William; William is taller than Charles; John is taller than Charles.

'Of two evils the less is to be preferred; occasional turbulence, therefore being a less evil than rigid despotism, is to be preferred to it.' (Whateley).

All fixed stars twinkle; yonder star twinkles; therefore it is fixed.

All that do not act foolishly are respectable; all fools act foolishly; no fools are respectable.

'Most men that make a parade of honesty are dishonest; this man makes a parade of honesty.' Can we conclude that he is dishonest?

Ill doers are ill dreaders. This man dreads evil, and is, therefore, a scoundrel.

All aristocracies are self-willed; some self-willed people are not cruel; some aristocracies are not cruel.

Some democracies are not persistent in their designs; the Government of the United States is a democracy; the Government of the United States is not persistent in its designs.

All plants contain cellular tissue; no animals are plants; no animals contain cellular tissue.

'I snatch at the conclusion that every eager desire is an evil thing; since I know that the desire of evil is evil, and that not a few eager desires have evil objects.' (Spalding).

A good marksman must have a steady hand; George has a steady hand; therefore, George is a good marksman.

Flotation is possible only in liquids, and so not possible in this water, which is frozen.

Poetry is not Science. The characteristics of Science are truth and generality, and Poetry possesses neither.

Nothing that is not possible for man to do has ever been done by man. Raising the dead is not possible for man, and, consequently, has never been done by man.

'If I know that Messieurs A. B. and C. are not only learned, men but also silly ones, will you allow me to draw any inference?' (Spalding).

Irrational prejudice is symptomatic of a weak mind, and we sometimes see it in very learned men. State this in syllogistic form, and draw the legitimate conclusion.

One who misapplies riches deserves poverty; which one who

is benevolent does not deserve. Is the legitimate conclusion consonant with fact?

‘If a rule never is, and a principle always is, a law admitting no exception, judge that a rule must be something different from a principle.’ (Spalding).

No branch of science can be made absolutely perfect, yet all branches of science are worthy of diligent culture. What inference do you draw from this?

‘What was it that first gained him the public ear? It certainly was not the pure Saxon-English in which his sentences are clothed, for, alas! we find that many writers who neglect their grammar even, secure an immense audience, to the delight of their publishers, and their own gratification.’

‘It has been supposed by some philosophers, that electricity is the real agent by which the nerves act upon the muscles. But there are many objections to such a view; and this very important one among the rest,—that electricity may be transmitted along a nervous trunk which has been compressed by a string tied tightly round it, whilst the passage of ordinary nervous power is as completely checked by this process as if the nerve had been divided.’

The following are examples of chains of reasoning, resolvable into consecutive syllogisms.

‘The concept ‘horse’ cannot, if it remain a concept, that is, a universal attribution, be represented in imagination; but except it be represented in imagination, it cannot be applied to any object; and except it be so applied, it cannot be realized in thought.’ (Hamilton).

‘But, to prove that moral sentiments are instinctive or inscrutable, it is boldly asserted, by the advocates of the hypothesis in question, that the moral sentiments of all men are precisely alike.

‘The argument, in favour of the hypothesis, which is raised on this hardy assertion, may be stated briefly in the following manner;—No opinion or sentiment which is a result of observation and induction is held or felt by all mankind. Observation and induction, as applied to the same subject, lead different men to different conclusions. But the judgments which are passed internally upon the rectitude or pravity of actions, or the moral sentiments or feelings which actions excite, are precisely alike with all men. Consequently, our moral sentiments or feelings were not gotten by our inductions from

the tendencies of the actions which excite them: nor were these sentiments or feelings gotten by inductions of others, and then impressed upon our minds by human authority and example. Consequently, our moral sentiments are instinctive, or are ultimate or inscrutable facts.' (Austin.)

'The general object which all laws have, or ought to have, in common, is to augment the total happiness of the community; and therefore, in the first place, to exclude, as far as may be, every thing that tends to subtract from that happiness: in other words, to exclude mischief. But all punishment is mischief: all punishment in itself is evil. Upon the principle of utility, if it ought at all to be admitted, it ought only to be admitted in as far as it promises to exclude some greater evil.' (Bentham).

'If our intellectual part is common, the reason also, in respect of which we are rational beings, is common: if this is so, common also is the reason which commands us what to do, and what not to do; if this is so, there is a common law also; if this is so, we are fellow-citizens; if this is so, we are members of some political community; if this is so, the world is in a manner a state.' (Marcus Antoninus). It is not to be supposed that all these transitions make distinct syllogisms; some are at best but immediate or equivalent transitions.

CHAPTER II.

RECENT ADDITIONS TO THE SYLLOGISM.

HAMILTON'S ADDITIONS.

SIR WILLIAM HAMILTON's extensions of the theory and the forms of the syllogism are chiefly based on the Quantification of the Predicate, and on the full development of the two modes of Quantity—Extension and Comprehension. He has also much criticism in detail on many parts of the syllogistic theory.

It has been seen (p. 86) that the thorough quantification of the predicate yields four new propositional forms, making eight in all. Two of these, the affirmative forms, 'All X is all Y,' 'Some X is all Y,' which are held by De Morgan and by Mill,

to be compound propositions, have been adopted by some other logicians, as Thomson ('Laws of Thought') and Spalding. The remaining two forms—the negative 'All X is not some Y,' 'Some X is not some Y' have been set aside as not occurring in actual instances.

The addition of two new forms greatly increases the number of possible syllogistic moods. By trying all the combinations of three propositions out of six, and by rejecting all that violate laws of the syllogism, and all that repeat others, Dr. Thomson makes out 22 moods in the First Figure, 20 moods in the Second Figure, 20 moods in the Third Figure; so that apart from the Fourth Figure, of which no account is taken, there are 62 moods. We give, as examples, some of the new moods.

U U U contains three universal affirmatives with universal predicates.

All Y is all Z

All X is all Y

All X is all Z

a syllogism, to which there is no counterpart in nature, unless the terms are merely different names for the same thing; as 'all water is all oxide of hydrogen.' We may find a proposition whose terms are of co-equal extent to constitute a major, (all matter is all gravitating things); but we shall probably never be able to couple with this a minor also co-extensive in its terms, if these terms really mean different things.

U E E is an example, constituting an exception to the canon requiring the minor in the First Figure, or normal deductive syllogism, to be affirmative.

All Y is all Z

All matter is all gravitating things

No X is Y

No mind is matter

No X is Z

No mind gravitates

Here the quantification of Z (universal) avoids illicit process of the major.

It is not pretended that any useful form grows out of these additions to the syllogistic moods; and even as a formal exercise, no one has thought it worth while to state them in full; far less to provide examples of them in the concrete.

Only Hamilton himself (followed by Professor Spencer Baynes) has endeavoured to enumerate the syllogistic moods growing out of the eight quantified propositional forms. He even gives the number variously. The earliest statement is thirty-six valid moods, for each figure (excluding the Fourth), that is, twelve affirmative, and twenty-four negative. Dr. Thomson has tabulated the forms, agreeing with Hamilton so

far, but deducting from Hamilton's complete list as useless though possible varieties, 14 moods in the first figure, 16 in the second, and 16 in the third. He thus reduces Hamilton's 108 moods to 62. In a later statement Hamilton gives 42 syllogisms, reducible to 21.

Syllogisms viewed either in Extension or in Comprehension. It is a great point with Hamilton to show that the common syllogism is defective, from not being expressed both in Extension and in Comprehension. He complains that all logicians, with the doubtful exception of Aristotle, have limited their consideration to reasoning as given in the quantity of Extension. He exemplifies the difference of the two syllogisms thus —

| <i>Extension.</i> | <i>Comprehension.</i> |
|--------------------|-----------------------|
| B is A | C is B |
| C is B | B is A |
| C is A | C is A |
| All men are mortal | Caius is a man |
| Caius is a man | All men are mortal |
| Caius is mortal | Caius is mortal |

In the first example the class 'mortal' contains under it the class man; in the second example, the attributes of 'man' contain in them the attribute 'mortal.'

The following is an example in *Celarent*,

| <i>Extension.</i> | <i>Comprehension.</i> |
|-------------------|-----------------------|
| No men are gods | Kings are men |
| All kings are men | Men are not gods |
| No kings are gods | Kings are not gods |

The second form (Comprehension) may be read thus:—

The attributes of a king contain the attributes of a man. The attributes of a man do not contain the attributes of a god. The attributes of a king do not contain the attributes characteristic of a god.

It is to be remarked, with reference to this scheme of double syllogisms, according as the terms are taken in extent, or in intent—breadth or depth—that the two modes express one and the same meaning; and that the really fundamental meaning is Intent, or the Connnotation of the Terms employed. The real meaning of the last example is, first, that the attributes connoted by the term, man, fail to accompany, or are incompatible with, the attributes connoted by the term, 'god' (major); that the attributes connoted by 'king' are accompanied with the attributes connoted by 'man.' The other form, however, falls readiest into common language, the form of Extension, that is, of inclusion or exclusion of

classes; men are out of the class of gods; kings are in the class men; therefore, kings are out of the class gods. This is a more concrete and intelligible form; still, it is not the contrast or the opposite of the other. We do not think of this form justly, correctly, unless we conceive the terms as determined by their *connotation*. The extent is bounded solely by the intent. It is not as if we had a complete list of men, and a complete list of kings, and saw the kings inserted among the men, while the list of men had nothing in common with the list of gods. This is the full and literal rendering of the reasoning in extension; and the very statement of it is enough to show that we do not reason so. When we speak of a class, we do so in a figurative manner; we suppose an actual array of individuals when there is no such array; there being only the defining mark, the connotation of them, to define them whenever they appear. The extent of 'man' is the imaginary aggregate of all objects agreeing in the marks connoted by the term, the defining characteristics of man; if we lose sight of this condition for a moment, we have nothing fixed in our grasp. Accordingly, comprehension is inseparable from extension in every case; it is an ever present fact, without our topsy-turvyng the syllogism, or constituting a parallel array of moods to match the moods in extension.

Hamilton's forms in comprehension depend solely on his introducing the idea of 'containing and contained' into the groups of attributes signified by the terms of the proposition. A king has more attributes than a man; the individual person 'Frederick the Second' has more attributes than a king. Thus, Frederick is the largest term, in point of number of attributes, man is the smallest. Hence we may, by straining a metaphor, apply the relation of whole and part, containing and contained, to this circumstance, as well as to the groups (in extension) men, kings, Frederick; and may carry the analogy so far as to construct syllogisms to match. But no new or distinct meaning is conveyed; and there is not even a more intelligible rendering of an old meaning.

Hamilton, in discussing the conditions of the Distinctness of Notions, remarks justly that the highest degree of distinctness cannot be attained without fixing the Comprehension, in other words, the meaning, definition, or connotation of the term. (Lectures on Logic 168). He remarks also that the quantity of Extension is a creation of the mind itself, and only created through, as abstracted from, the quantity of comprehension;

whereas the quantity of comprehension is at once given in the nature of things (p. 218). All which tends to the conclusion that the comprehension is what we think of in a notion; and consequently the comprehension cannot be left out of the account in any syllogistic form. It is the power behind the throne, even when extension is the ostensible reigning circumstance.

In objecting to the Fourth Figure, Hamilton grounds his dislike on the circumstance, that the premises proceed in the whole of comprehension, while the conclusion is drawn in the counter whole of extension. He explains the matter thus. The scheme of the Figure is—

P is M

M is S

S is P

Now in the premises P is contained under M; and M contained under S; whence in the conclusion we should expect P to be contained under S. In this, however, we are disappointed; for the reasoning suddenly turns round in the conclusion, and affirms S as a part of P. [Not strictly correct; for S is qualified by 'some,' which may still leave it the larger term; 'Some S is P.'] If we had an affirmative syllogism in the form

All P is M All kings are men

All M is S All men are fallible

All S is P All fallible beings are kings

we should have an illegitimate inference; which might no doubt be evaded if the conclusion could be read thus—

All the *attributes* of fallible beings are *contained* in the attributes of Kings.

But no one ever reads the figure in this way.

DE MORGAN'S ADDITIONS.

We have seen Mr. De Morgan's views as to Terms, and his enumeration of Fundamental Propositions. Before proceeding to view his enlargements of the Syllogism, we shall advert to his remarks on the COPULA.

He complains that the 'is' of logicians is not confined to one strict meaning. It professes to be a word of the highest abstraction, a formal mode of joining two terms, carrying no meaning, and obeying no law, except such as is barely necessary to make the forms of inference hold good. 'X is Y' commits us to nothing specific. Yet, at times, logicians employ it in the sense of identity. The best description of its employ-

ment, he considers to be—‘agreement in some understood, and, for the occasion, unvarying particular.’

He supposes that a copular *symbol* had been used, instead of ‘is;’ the effect of which would have been to stamp upon the copula the character of an abstraction, as is done by the use of symbols, X, Y, Z, for terms. Had such a symbol been used, the *copular conditions* would have been stated. These are two in number. The first is *transitivity*; meaning that if X stands in a certain relation to Y, and Y in the same relation to Z, X stands in the given relation to Z. Very many copulæ show this transitive relation;—is,—rules,—lifts,—draws,—leads to,—is superior to,—is ancestor to,—is brother of,—joins,—depends on,—is greater than,—is equal to,—is less than,—agrees with (in a given particular), &c.

The second condition is *convertibility*, in which the relation is its own correlation; whatever X is to Y, Y is to X. In a certain number of the foregoing examples, there occur convertible relations; is,—is brother of,—joins (if a middle verb),—is equal to,—agrees with. There are cases of convertibility without the transitive character; converses with,—is in the habit of meeting,—is cousin of,—is in controversy with, &c.,

Again, there are copula not convertible, but *correlative*; A gives to B; B receives from A. These forms also are duly reasoned upon; and syllogisms might be constructed accordingly. Every X gives to a Y; Some Xs give to no Ys; No X gives to a Y; Every X receives from a Y; Some Xs receive from no Ys,—are examples of the propositional forms. They are all capable of conversion, by substituting the correlative copula.

The admission of Relation in general, Mr. De Morgan contends, and of the composition of relation, makes logic more in alliance with ordinary thinking. The reduction of all relations by ‘is’—‘mind acts on matter, mind is a thing acting on matter,—is a systematic evasion, hostile to the progress of the science.

Logicians are aware that the form ‘A equals B, B equals C, therefore A equals C’ is not reducible to the syllogism. So with the relation of ‘greater than,’ in the argument *a fortiori*. Yet, to the ordinary mind, these inferences are as natural, as forcible, and as prompt, as the syllogistic inference. Mr. De Morgan, therefore, would propose to include all such forms in one sweep by a generalized *copula of relation*, which would be formally embodied and symbolized in propositions. Thus—

Every X has a relation to some Y

Every Y has a relation to some Z

from which the inference would be that 'Every X has a compound relation to some Z;' the compound of the relations X to Y, and Y to Z. Under this form, we reason, John can control Thomas; Thomas can control William; John can control William. Under the general and comprehensive copular relation, specific modes might be developed for specific purposes. The Logical copula in common use is the equivalent of 'fastened to,' 'connected with,' 'co-exists with,' and may be considered for logical purposes the most important. The copula of equality and inequality is developed in Mathematics, and an inference according to it would probably be called a mathematical inference.

The converse copular relation, 'causes,' would be singled out on account of its great importance:—A causes B, B is caused by A. We practically construct syllogisms from these propositions, without passing through our minds the formal transformation to—A is the cause of C.

These remarks of Mr. De Morgan's are undoubtedly just and cogent; and they are highly valuable in the way of emancipating the student from the Aristotelian limits, as well as for pointing out the vagueness and vacillation of the ordinary copula. Still, we could hardly afford the labour of following out the technical developments of half-a-dozen distinct forms of copula. It is well to see that such developments are not merely competent in themselves, but needed to formulate the whole compass of our habitual thinking and reasoning. Being, however, aware of this fact, we must be content with constructing one scheme adapted to the most useful and most frequently recurring relationship; which scheme we should then regard as an example of the rest, one out of many. Any one having Mr. De Morgan's genius for the construction of forms might do well to develop a variety of copular relations; from these such selections might be made as would extend the inferential grasp of the ordinary student.

Mr. De Morgan's Extensions of the Syllogistic forms are avowedly based upon the full recognition of contraries, as laid out in his scheme of eight fundamental propositions. Also, by providing symbols for contraries he can exhibit all denials as assertions; No X is Y, is All X is y ($U-Y$). Hence, the unit syllogism may be represented in an affirmative form—'If an X be a Y, if that same Y be a Z, then the X is a Z.'

All syllogisms are derivable from the following combinations of Premises :—

(1) All Xs are Ys, and all Ys are Zs. The conclusion is All Xs are Zs; the unit syllogism. This is the inversion of the Aristotelian order of premises, but it is in the author's view the proper and the natural order.

(2) Some Xs are Ys, all Ys are Zs; some Xs are Zs. The unit syllogism is here, as it were, cut down to the form,—‘as often as there are Xs in the first premise, there are in the conclusion.’

(3) Some Xs are all Ys, some Ys are Zs; conclusion—some Xs are Zs. In point of form, this is the previous case inverted. The universal middle term (all Ys) is transferred from the second premise to the first.

(4) Some Xs are all Ys, All Ys are Zs; Some Xs are Zs. Here, although there is an additional universal middle, all Ys. occurring in both premises, there is no stronger conclusion than in the two preceding cases, where the middle term is universal (or distributed) only once.

These are all the possible couples of affirmative premises apart from any cognisance of contrary terms. Now, all negations may be rendered as affirmations about contraries; and therefore the application of these cases to all combinations of propositions, direct or contrary, will give all possible valid syllogisms.

Taking X, Y, Z, and their contraries x, y, z, there are eight combinations of threes :—X Y Z, x Y Z, \bar{x} y Z, x y z, X Y z, X y Z, X y z, x Y Z. To each of these the four modes of inference can be applied; and when x, y, z, are read as the contraries of X, Y, Z, we obtain the proper expression of the syllogism. Thus, the first or unit syllogism, applied to x y Z, gives Every x is y, Every y is Z; therefore, Every x is Z. This unfolded, by giving the equivalents of the contrary terms x, y, in the forms X, Y, the whole syllogism may be read thus :—

Every x is y (All not-X is not-Y) is the same as No Y is not X, or Every Y is X, or Some Xs are all Ys.

Every y is Z (Every not-Y is Z) is the same as Everything is either Y or Z (one of De Morgan's new propositional forms).

In like manner, the conclusion Every x is Z, (Every not-X is Z) is Everything is either X or Z. The syllogism then is :—

Some Xs are all Ys (Every Y is X).

Everything is either Y or Z.

Everything is either X or Z.

A syllogism not in the Aristotelian figures. From the very

wide compass of the form, Everything is either Y or Z, there can be few applications of such a syllogism.

Some extended things are all material things.

Everything is either material or pertaining to mind.

Everything is either extended or pertaining to mind.

The remaining seven forms being expressed and unfolded in like manner, there would arise the *eight forms of universal syllogism*, that is *universal premises with universal conclusion*.

Again, apply case second to the same eight forms—Some Xs are Ys, all Ys are Zs; some Xs are Zs; and there emerge *eight minor-particular syllogisms, particular conclusion with the minor (or first) premise particular*.

Apply case third—Some Xs are all Ys, some Ys are Zs; some Xs are Zs—and we have *eight major-particular syllogisms, particular conclusion with the major (or second) premise particular*.

Apply case fourth—Some Xs are all Ys, All Ys are Zs, Some Xs are Zs—and we have *eight strengthened particular syllogisms, universal premises with particular conclusion*. By a strengthened syllogism, the author means one whose premises are stronger than they need be to bear out the conclusion.

The above 32 forms are those that give inference, out of 64 possible combinations of the premises. The remaining 32 forms could be drawn out by representing the eight propositional arrangements, $X\ Y\ Z$, $x\ Y\ Z$, &c., in four varieties of premises, which the author states. Thus: (1) Some Xs are some Ys, Some Xs are all Ys; (2) All Xs are some Ys, Some Xs are some Ys; (3) Some Xs are some Ys, Some things are neither Xs nor Ys; (4) Some Xs are Ys; All Xs are not some Ys. From none of these combinations of premises could any inference be drawn.

The test of validity, and the rule of inference, the author expresses thus:—

There is inference (1) When both the premises are universal. (2) When, one premise only being particular, the middle term has different quantities in the two premises. Either of these cases happening, the conclusion is found by erasing the middle term and its quantities. Premises of like quality give an *affirmative* conclusion; of *different* quality, a *negative*. A universal conclusion follows only from universals with the middle term differently quantified in the two. From two particular premises nothing follows.

A particular premise having the *concluding term strengthened*

(that is, made universal), the conclusion is also strengthened, and the syllogism becomes universal; for example, *Darii*, by this process, would become *Barbara*. With the *middle* term strengthened, the conclusion is not strengthened, and there being, therefore, a surplus of affirmation in the premises, the syllogism forms what the author calls a *strengthened particular syllogism*. Thus, *Darapti*, in the third figure—

All Y is Z

All Y is X

Some X is Z—

has the middle term universal in both premises, when once is enough, there would be inference with 'Some Y is X' in the minor. *Felapton* and *Fesapo* are other examples.

A different case is exemplified in *Bramantip*. The two universals—'All Z is Y, All Y is X,' yield the universal 'all Z is X,' which, for the sake of a different order of the terms in the conclusion, is converted and weakened into the particular 'Some X is Z.' This is termed by the author a weakened universal.

Each form of proposition has corresponding to it certain *opponent* forms. Thus, if the propositions A, B, gives C, they cannot give c (the contrary of C). Hence A and c being true, B is false or B true; that is A, c, give B, that is to say, *either premise joined with the contrary of the conclusion gives the contrary of the other premise*. Thus, there are two opponent forms to every syllogism. And the syllogisms may be so grouped in threes, that each one of any three may have the two others for opponents. *Barbara* has, for opponent forms, *Baroko* and *Bokardo*.

Mr. De Morgan considers it of importance to remark that the adjective for expressing universal quantity—'All' means two things, which should be kept distinct. It may be 'All' collectively, the entire collection or aggregate of individuals; this he calls the *cumular* form; and it may be 'all' distributively, in the sense of 'every one,' or 'any one,' however taken, which he calls the *exemplar* mode. He holds that the language of Aristotle, and of his immediate followers, was exemplar and not cumular; *πᾶς ἀνθρώπος*, he contends, is *each* or *every* man, not *all* man. 'All man,' as a comprehensive genus, has parts,—for example, the several species or varieties of men; 'every man' has no parts, but makes assertions about every individual of the genus man.

The exemplar mode is that used in geometrical proof. A proposition in Euclid assumes *some one* case, and the demon-

stration is such that nothing prevents the one chosen from being *any one*. It would be useful in geometry, to admit the form 'any one X is any one Y.'

In negation, the exemplar form is needed. 'All men are not fishes,' does not deny the proposition, 'All men are fishes.' The denial would, however, be given in 'Every man is not any fish.*'

Properly speaking, the cumular proposition can be founded only through exemplars; hence the exemplar precedes in the order of thought; a circumstance justifying its adoption as the basis of a logical system. According to it, *quantity* is *mode of selection by example*; universal is replaced by *wholly indefinite*; particular by not *wholly indefinite*. The forms of the propositions would be modified thus:—

Any one X is any one Y. X and Y singular and identical.

Some one X is not some one Y. Either X not singular, or Y not singular; or if both singular, not identical.

Any one X is some one Y. All Xs are some Ys.

Some one X is not any one Y. Some Xs are not (all) Ys.

Some one X is any one Y. Some Xs are all Ys.

Any one X is not some one Y. All Xs are not some Ys.

Any one X is not any one Y. All Xs are not (all) Ys.

Some one X is some one Y. Some Xs are some Ys.

The 'Numerically Definite Syllogism' is a scheme of inference which supposes exact numbers to be given.

If in 100 instances of any thing, 70 are Xs, and 50, Ys, then at least 20 Xs must be Ys. The author develops at great length a symbolical scheme founded on this assumption.

Syllogisms with numerically definite quantity occur rarely, if ever, in common thought. But it is not unfrequent to find forms where the number of instances of one term is the whole number of instances of the other term;—'For every Z there

* Mr. Mill, in a controversial note to his chapter on the Functions of the Syllogism, makes the following remark:—The language of ratiocination would, I think, be brought into closer agreement with the real nature of the process, if the general propositions employed in reasoning, instead of being in the form All men are mortal, or Every man is mortal, were expressed in the form *Any man is mortal*. This mode of expression, exhibiting as the type of all reasoning from experience "The men A, B, C, &c. are so and so, therefore *any* man is so and so," would much better manifest the true idea—that inductive reasoning is always, at the bottom, inference from particulars to particulars, and that the whole function of general propositions in reasoning, is to vouch for the legitimacy of such inferences.

is an X. that is Y ; some Zs are not Ys ;' ' For every man in the house there is a person that is aged ; some of the men are not aged ;' from which it follows, but not by any common form of syllogism, that ' some persons in the house are not men.'

To this case the author applies the designation 'syllogism of *transposed* quantity.' Of terms in common use the only one that gives syllogisms of this character is 'most :—' Most Ys are Xs ; most Ys are Zs ; therefore *some* Xs are Zs.'

• Adverting to the distinction of Figure, he styles the First the figure of *direct transition* ; the Fourth, which is nothing but the first with a *converted* conclusion, the figure of *inverted transition* ; the Second, the figure of *reference to* (the middle term) ; the Third, the figure of *reference form* (the middle term). Apart from the conversion of the conclusion, the Fourth Figure is the most natural order, as it takes up what was left off with—' X is in Y, Y is in Z, therefore X is in Z : ' this is the first figure, according to the simplest arrangement of the premises.

• In the author's system, however, Figure attains importance only through a wider view of the *copular relation*.

Mr. De Morgan compares his system with the Aristotelian, of which he regards it as an extension, through the single device of adding contraries to the matters of predication. (Hamilton also claims to extend Aristotle, but on a different principle). Accordingly the Aristotelian syllogisms may be all collected from the preceding system, by the following modifications. 1. The exclusion of all idea of a limited universe, of contrary names, and of the propositions, ' Every thing is either X or Y,' ' Some things are neither Xs nor Ys ' 2. The exclusion of the form of conversion, ' Some Xs are all Ys.' 3. The exclusion of every copula except the transitive and convertible copula. 4. The regarding of the identical pairs—No X is Y, No Y is X, and Some X is Y, Some Y is X—as distinct propositions of themselves determining distinction of figure and mood ; as *Celarent* and *Cesare*, *Ferio* and *Ferison*, &c. 5. The introducing of the distinction of figure. 6. The writing of the major and minor propositions first and second, instead of second and first.

Farther, in the Aristotelian scheme, there are four fundamental syllogisms in the first figure, each of which has an opponent in the second, and an opponent in the third. The opponents of *Barbara* are *Baroko* and *Bokardo*. There are three fundamental syllogisms in the fourth figure (*Dímaris*,

Camenes, *I'esison*), each of which has the two others for opponents. Altogether there are fifteen fundamental syllogisms. The remaining four are—three strengthened particular syllogisms, *Darapti* (III), *Felapton* (III), *I'esapo* (IV), and one weakened universal; *Bramantip* (IV).

The Aristotelian rule that the middle term must be distributed once fails with the introduction of contraries. The rule to be substituted is—All pairs of universals are 'conclusive, but a universal and a particular require that the middle term should also be a universal and a particular,—universal in one premise and 'particular in the other.

The rule that when both premises are negative, there is no syllogism, also fails. In the system completed by contraries, there are eight such syllogisms; as many, in fact, as with premises both affirmative. But in these cases, as before remarked, the premises are not both negative in reality.

Again, on the rule 'that two particular premises can give no conclusion,' the author brings forward as a legitimate inference, 'Most Ys are Xs, most Ys are Zs, therefore some Xs are Zs; most men wear coats, most men wear waistcoats, therefore some men wear both coats and waistcoats.' He develops this form at length into a symbolical scheme, under the name of 'The numerically definite syllogism.'

Mr. De Morgan's system, on the whole, is characterized by an immense multiplication, not only of symbolical forms, but of *verbal designations* for the relationships growing out of the syllogism.

BOOLE'S ADDITIONS.

The late Professor Boole, of Cork, published two works on Formal Logic. The first and smaller, entitled—'The Mathematical Analysis of Logic,' comprised an Algebraic rendering of the syllogism, showing how all the moods might be symbolically deduced. The second and larger work, entitled—'An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities,' takes a much wider sweep, and is an entirely new application of the symbolical methods of Algebra, to Inference, both Immediate and Mediate; the largest share of attention being given to the first, or the so-called Immediate Inference. The author also extends the same nomenclature and handling to Probabilities.

Besides the novel employment of symbolical processes of the Algebraic kind, the work is intended to bear fruit in other

ways. In using the title 'Laws of Thought,' the author indicates that one purpose of his theory of Reasoning is to throw light upon the workings of the Intellect. • He considers that our views of the Science of Logic must materially influence, perhaps mainly determine, our opinions upon the nature of the intellectual faculties. For example, whether reasoning consists merely in the application of certain first or necessary truths, originally imprinted on the mind, whether the mind is itself a seat of law [whatever that may mean], or whether all reasoning is of particulars, concerns not Logic merely, but also the theory of the intellectual faculties. It cannot be said, however, that the author has been able to decide which alternative is the correct one.

He farther proposes to elucidate the subtle connexion between Logic and Mathematics; how far a common theory is applicable to both kinds of reasoning, and how far the likeness fails. He holds that the ultimate laws of Logic are mathematical in their form, that they are, except in a single point, identical with the general laws of Number. The exhibition of Logic in the form of a Calculus is not arbitrary: the ultimate laws of thought render that mode possible, and forbid the perfect manifestation of the science in any other form. It is not of the essence of Mathematics to be conversant with the ideas of number and quantity. The author does not design to supersede, by symbolic processes, the common forms of reasoning; nevertheless, cases may arise where the value of scientific procedure, even in things confessedly within the scope of ordinary reasoning, may be felt and acknowledged.

The author's scheme starts with the consideration of Language as an instrument, not of communication merely, but of Reasoning; it being his intention to substitute, for ordinary language, a set of symbols adapted to perform this function in a more effective manner.

The signs composing Language, with a view to Reasoning especially, are characterized in the following definition:—'A sign is an arbitrary mark, having a fixed interpretation, and susceptible of combination with other signs in subjection to fixed laws dependent upon their mutual interpretation.' The first part is obvious; a sign, in its primary invention is purely arbitrary; 'house' and 'domus' are equally good for the purposes of language. It is also obvious that each sign should possess a fixed interpretation, that there should never be any ambiguity of meaning. Ordinary language is greatly liable to

this infirmity; hence, one of its defects as an instrument of reasoning. Lastly, signs must be susceptible of combination with other signs, which combinations must have fixed laws depending upon their mutual interpretation.

The author proceeds to explain his artificial symbols for superseding, by a higher mechanism, the vocables of our ordinary speech. The symbols, and their connecting signs of operation, are borrowed from Algebra, and are manipulated by the algebraic processes, after allowances are made for the difference between the material of Logic, and the material of Mathematics (Number and Quantity).

All the operations of Language, as an instrument of Reasoning, may be conducted by a system of signs composed of the following elements:—

First, Literal symbols, as x , y , z , &c., representing things as subjects of our conceptions. For the object 'man' we may use x , for a 'brute,' y , for the quality 'living,' z , and so on.

Second. Signs of operation, as $+$, $-$, \times , standing for the operations whereby conceptions are combined, or, when combined are resolved into their elements; 'men and brutes' may be represented by $x + y$.

Third. The sign of identity $=$.

These symbols of Logic are used according to definite laws, partly agreeing with, and partly differing from, the laws of the corresponding symbols in the science of Algebra.

The first class of symbols above given are the appellative or descriptive signs, expressing either concrete things, or the qualities of things; that is to say, they are the equivalents of the two appellative parts of speech, the Noun and the Adjective. Thus, let x denote 'men,' or all men; and let y denote the adjective good; then all good men would be expressed by some suitable combination of x and y . Now the suitable combination, for the case of a thing qualified by an attribute, or of two or more co-inhering attributes is a product $x \times y$, or xy . Why this, and not the sum $x + y$, is the proper symbol, the author does not specifically explain; the means, as in other symbolical sciences, are left to be justified by the end, namely, arriving at true results. So if x stands for 'white' or 'white things,' y for sheep, xy stands for 'white sheep;' and if z stands for 'horned,' zx will represent 'horned white sheep.' In this symbolism, the order of the symbols is unimportant, just as the order of the adjective and the substantive is indifferent as regards the meaning; 'good man,' 'vir bonus' are equally accepted by the mind to suggest that

the conception 'mau' is to be limited by the conception 'good.' Hence we may use at pleasure $x y$, and $y x$; $x y z$, and $z y x$, &c.

It is a law of speech that an appellative gains nothing (except perhaps rhetorically) by repetition or duplication; 'good, good,' is the same as good; 'horse, horse,' is the same as horse. To adapt this to symbols, $x x$ would amount to no more than x ; that is, using $=$ (as in Algebra) for equivalence, or identity, $x x = x$. Here Logic and Algebra are at variance, and the methods of manipulating logical symbols must vary accordingly. The author shows that the form $x x \doteq x$, or $x^2 = x$, has still deeper meanings.

Next as to signs for collecting parts into a whole (quantity in extension) or for separating a whole into parts. These correspond to the conjunctions 'and,' 'or,' in common speech—'trees *and* minerals;' 'barren mountains, *or* fertile vales.' The sign of addition is now used; let x be 'trees' and y 'minerals;' the conjoined expression is $x + y$. This employment of the sign is so closely allied to addition in arithmetic, that it may be worked upon the same principle. Again, let x stand for men, y for women, and z for European; then 'European men and (European) women' would be represented by $z (x + y) = z x + z y$.

Addition implies subtraction. 'All men except Europeans' will be expressed by $x - y$. 'White men except white Asiatics' (x men, y Asiatics, z white),

$$z (x - y) = z x - z y$$

With a view to Propositions, it is necessary to consider the rendering of the copula. For this purpose all propositions have to be reduced to the form 'is' or 'are;' 'Cæsar conquered the Gauls,' must be resolved into 'Cæsar *is* he that conquered the Gauls.' This is the copula of identity, the most generalized form of relationship of subject and predicate. It may be expressed by the symbol $=$; and the meaning so far coincides with the Algebraic meaning, that the Logical equation is little different from the Algebraic equation.

Take the Proposition, 'The stars are the suns and the planets.' Let stars be represented by x , suns, by y , and planets, by z ; then,

$$x = y + z$$

Whence we can deduce,

$x - y = z$ (The stars, except the suns, are planets),
or, $x - z = y$ (The stars, except the planets, are suns).

Thus, in the Logical equation, we may apply the mathe-

mathematical axioms 'equals added to equals give equal sums;' 'equals taken from equals give equal differences.'

If two classes of things, x and y , be identical, that is, if all members of the one are members of the other, then such members of the one class as possess a given property, z , will be identical with the members of the other that possess the same property. Hence, if we have the equation

$$x = y:$$

then, whatever class or property z may represent, we have also

$$zx = zy.$$

In point of form, this coincides with the algebraic law—that both members of an equation be multiplied by the same quantity, the products are equal.

The analogy, however, does not extend to division. For, supposing the members of a class x , possessing the property z , are identical with the members of a class y , possessing the same property, it does not follow that the members of the class x universally are identical with the members of the class y . Hence, it cannot be inferred from the equation

$$zx = zy,$$

that the equation

$$x = y$$

is also true. Thus, the process of division, as applied to equations in Algebra, has no formal equivalent in Logic. Multiplication sufficiently represents the combination or composition of conceptions, but division does not appear to represent their decomposition or abstraction. The want of analogy on this point, however, is not total. Even in Algebra, the rule of division does not hold throughout; for example, it does not apply when the divisor is $z = 0$. Through this one loophole, the author is able to restore the consistency of the algebraical and the logical processes.

Reverting to the equation

$$x^3 = x$$

he remarks that only two values of x will comply with it; namely, 0 and 1. For $0^3 = 0$, and $1^3 = 1$; and of no other numbers is the relation true. Hence, in an Algebra, whose symbols x , y , z , &c., never knew any values but 0 and 1, the laws of operation would coincide with the laws of operation in Logic. The two sciences are divided by no other difference than the manner of interpretation.

In chapter III., Boole professes to derive the laws of the symbols of Logic, above assumed, from the laws of the opera-

tion of the mind. He proceeds thus:—In every discourse there is a limit to the subjects considered; in other words, a *universe*. [He is here at one with De Morgan]. Thus the term 'men' is used with reference to a certain implied extension, on the part of the speaker; it may be all men whatsoever; or it may be a more limited universe, as civilized men, men in the vigour of life, and so on. The term 'men' raises in the mind of the hearer the beings so intended to be comprised. Let us next consider the employment of an adjective in addition. Suppose 'men' to be spoken of in the widest sense, the universe 'all men;' then the application of the adjective 'good' prescribes the operation of selecting from the universe all objects possessing the further quality 'good;' such selection corresponds to the combination—good men. Thus, the office of an adjective is not to add the quality, 'good' for instance, to all the universe, men, but to *select*, from the universe, individuals according to the idea prescribed in the word. The intellectual faculties employed in these successive operations may be supposed to be those denominated Conception or Imagination, and Attention; or perhaps the entire act may be summed up in one function of Conception. Each step in the process may be characterized as a *definite act of conception*.

Now, the symbol above adopted exactly corresponds to this operation. The symbol x directs attention upon a certain universe, men for example; the symbol y , good or white, directs us to search that universe for individuals owning the property named; and the combination $y x$, or $x y$, expresses the selection—good men or white men. This symbol will not fall under the relations expressed by a sum; its meaning is a group qualified by the conjoined conceptions x and y , not an aggregate made up by adding the universe x to the universe y . In this way does Boole consider that he has established his positions: (1) that the operations of the mind are subject to general laws, and (2) that these laws are mathematical in their form; whence the laws of the symbols of Logic are deducible from the operations of the mind in reasoning.

He then proceeds to determine the logical value and significance of the symbols 0 and 1, to which quantities Algebra has to be cut down, in order to become Formal Logic. The symbol 0 corresponds to Nothing; the symbol 1 corresponds to the universe of discourse. Nothing and Universe are the two limits of extension—none and all. Whatever the class y may be, the individuals common to it and to the class 0, or Nothing, are Nothing or none. That is,

$$0 \times y = 0, \text{ or } 0 y = 0$$

Again, the symbol 1, satisfies the law of equation,

$$1 \times y = y, \text{ or } 1 y = y$$

whatever y may represent. The class represented by 1, therefore must be 'the Universe,' the only class containing all the individuals that exist in *any* class.

Now as to contraries. If x represent any class of objects, $1 - x$ will represent the contrary, or supplementary class, what remains when x is withdrawn from the Universe of discourse 1. If x be 'men' in the universe 'animals,' $1 - x$ is the not-men, the remaining members, or the brutes. This coincides with De Morgan's symbolism, $U - x$ for the contrary of x .

The author next offers from his fundamental logical equation, $x^2 = x$, or $x - x^2 = 0$, a formal proof of the Law of Contradiction, thus:—The equation admits of the form

$$x (1 - x) = 0$$

which, being interpreted according to the meaning of the symbols, is that a class determined at once by x , and by its contrary $1 - x$, is the same as 0 or Nothing; that is, does not exist.

Advancing farther into the consideration of Propositions (chap. IV.), the author divides these into 'primary' or simple, and 'secondary' or complex; the one relating to things, the other to propositions. Under the last named class are included hypotheticals, &c. He begins by propounding a general method for expressing any 'term' that may enter into a primary proposition. The method is merely the application of his symbols as already explained. Thus, let x represent opaque substances, y polished substances, z stones; then

$$x y z = \text{opaque polished stones.}$$

Now as $1 - z$ represents substances that are the contrary of stones, or are not stones,

$x y (1 - z) = \text{opaque polished substances that are not stones;}$
So

$x (1 - y) (1 - z) = \text{opaque substances, not polished, and not stones.}$

Again, for the case of collections of things,—or objects conjoined by 'and,' 'or,'—the sign of addition must be added, as above explained. The sign 'or' gives a disjunctive form; all x 's are either y 's or z 's; and this has two meanings not discriminated by the use of 'or,' but differently rendered in the formula. It is a question whether x may, or may not be both y and z . 'He is either a rogue or fool;' he may or may not be both, so far as this expression goes, although the more

usual rendering would be 'not both.' The two ways of symbolic expression are the following. (1) Things that are either x 's or y 's, are things that if x 's are not y 's, and if y 's are not x 's: that is

$$x(1-y) + y(1-x).$$

(2) Things that are either x 's, or if not x 's, then y 's.

$$x + y(1-x).$$

This admits the supposition of being both x and y , a supposition more explicitly given in the enlarged equivalent form.

$$xy + x(1-y) + y(1-x),$$

where we have all three alternatives: xy expressing the concurrence of both x and y . If he is not a rogue he is a fool, x fool, y rogue, $x(1-y)$; if he is not a fool he is a rogue, $y(1-x)$; he is a fool and a rogue together, xy .

To take a more complex example, exhibiting the full power of the method; let

$$x = \text{hard}, y = \text{elastic}, z = \text{metals};$$

and we shall have the following results:

$$\text{non-elastic metals} = z(1-y).$$

Elastic substances, together with non-elastic metals, $y + z(1-y)$.

Hard substances except metals, $x - z$.

Metallic substances, except those neither hard nor elastic,

$$z - z(1-x)(1-y) \text{ or } z \left\{ 1 - (1-x)(1-y) \right\}.$$

To take a still more complicated example: 'Hard substances, except such (hard substances) as are metallic and non-elastic, and such (hard substances) as are elastic and non-metallic.' Hard substances being represented by x ; substances hard, metallic, and non-elastic, are $xz(1-y)$; substances hard, elastic, and non-metallic, are $xy(1-z)$, and the whole expression is

$$x - \left\{ xz(1-y) + xy(1-z) \right\} \text{ or } x - xz(1-y) - xy(1-z).$$

Such is the expression of Terms. To form Propositions, the sign $=$ is used for the copula of identity. Thus, to express identity between 'Fixed Stars' and 'Suns,' or to express that 'All fixed stars are suns,' and 'All suns are fixed stars,' [Hamilton's universal with universal predicate],

$$x = y.$$

This is the form applicable to the verbal proposition or definition; and the author exemplifies it by such. For example, Senior's definition of wealth, as consisting in things transferable, limited in supply, and either productive of pleasure

Suppose the case, 'No men are perfect beings,' a universal negative. Here, we make an assertion to the effect that '*all men*' are 'not perfect beings.' The meaning may then be expressed thus:—*All men* (subject) *are* (copula) *not any part of perfect* (predicate). Let y represent 'men,' and x 'perfect beings.' 'Not perfect beings' are represented by the negative form $1-x$; and 'some not perfect beings,' by this form, qualified by the sign of particularity, v . Hence, the equation is

$$y = v(1-x).$$

Thus, to express the form No xs are ys , we have to convert it into 'All xs are not (any part of) ys .'

A particular negative proposition, 'some men are not wise,' is resolvable into 'some men' (subject) 'are' (copula) 'not wise' (predicate). Putting, then, y for 'men,' x for 'wise,' and v for an indefinite containing some individuals of the class qualified by it, we have for 'some men,' vy , for 'not any part of the wise,' $v(1-x)$, or the equation

$$vy = v(1-x).$$

So much for the symbolical expression of primary or simple propositions. It is next to be seen how these forms are turned to account in furnishing immediate inferences, or in exhausting all the equivalent propositional forms of each; in which operation the author principally expends the force of his method.

With this view, permission must be given to work the several equations after the algebraical model, with the restrictions already stated. The reader must be satisfied from the explanations afforded that the signs used have the same force in Logic as in Algebra. The conditions of valid reasoning are then those three:—First, that a fixed interpretation be assigned to the symbols; secondly, that the formal processes of solution or demonstration be conducted in obedience to the laws laid down as to the meanings of the signs of operation; thirdly, that the final result be interpreted in the same way as the original data. Having once clothed the logical meaning in the algebraic dress, the author claims to proceed exactly as if he had to deal with an algebraic equation wherein the symbols have only the two meanings 0 and 1.

The exhaustive renderings of each proposition are to be gained by a process of 'development,' which is explained at length, and is strictly after the manner of Algebra, with the conditions of value specified. The skeleton of the form of development is furnished from these considerations:—Suppose we are considering a class of things with reference to the point

whether its members possess or do not possess a property x ; as animals, with reference to humanity. Suppose next that the members possessing the property x , possess also a property u ; and that the members not possessing the property x are subject to a condition v . On these suppositions the class in its totality is represented by

$$u x + v (1 - x).$$

Any function of x , $f(x)$, wherein x is a logical symbol, susceptible only of the values 0 and 1, is said to be developed, when it is reduced to the form $a x + b (1 - x)$, a and b being so determined as to make the result equivalent to the function whence it is derived. The following out of this development is purely algebraical, and occupies a good many pages of the work. To a student versed in ordinary Algebraical equations, the whole is sufficiently intelligible. We shall here indicate merely the results and applications. The following is given as an example. It is a definition with two defining marks. 'Clean beasts are such as both divide the hoof and chew the cud.'

Let x = clean beasts,

y = beasts dividing the hoof,

z = beasts chewing the cud.

The definition will then be represented by the equation

$$x = y z,$$

which may be reduced to the form

$$x - y z = 0.$$

Here a function of x , y , and z , namely $x - y z$ has to be developed according to the methods laid down. As a specimen, we may transcribe the development;

$$0xyz + xy(1-z) + x(1-y)z + x(1-y)(1-z) - (1-x)yz + 0(1-x)y(1-z) + 0(1-x)(1-y)z + 0(1-x)(1-y)(1-z).$$

Now all those terms that are multiplied by 0 necessarily vanish and the remaining terms are

$$xy(1-z)=0, xz(1-y)=0, x(1-y)(1-z)=0, (1-x)yz=0.$$

Which equations all express the denial, or nothingness, of the combinations given in the left side of each. Thus $xy(1-z) = 0$ means that there cannot be beasts that are clean (x) and that divide the hoof (y), and that do not chew the cud ($1-z$). So the last of the four, $(1-x)yz = 0$, indicates that there are no beasts unclean ($1-x$) and yet dividing the hoof (y), and chewing the cud (z).

These equivalent forms are somewhat obvious in themselves without the aid of analysis; but the author evolves more complicated equivalents, such as these:—'Unclean beasts are

all that divide the hoof without chewing the cud, all that chew the cud without dividing the hoof, and all that neither divide the hoof nor chew the cud.' The reader may be curious to see the corresponding equation :—

$$1 - x = y(1 - z) + z(1 - y) + (1 - y)(1 - z).$$

It is obvious, from this instance, that, out of a definition containing three or four defining marks (Senior's definition of wealth, for example), a great many equivalent forms are derivable. Whether there be any important form that the unassisted mind might not evolve, is not quite apparent. It is possible, however, that cases might arise where the symbolical method would yield equivalents too recondite for an intellect with only the ordinary logical training.

The author extends his analysis so as to comprise a more difficult order of examples, typified thus. Suppose the analysis of a particular class of substances has conducted us to the following general conclusions, namely :—

First. Wherever the properties A and B are combined, either the property C or the property D is present also ; but they are not present jointly.

Secondly. Wherever B and C are combined, A and D are either both present or both absent.

Thirdly. Wherever A and B are both absent, C and D are both absent also ; and *vice versa*, where C and D are both absent, A and B are both absent also.

Let it then be required from these conditions to determine what may be concluded in any particular instance from the presence of the property A, with respect to the presence or absence of the properties B and C, paying no regard to the property D. The working of the corresponding equations leads to this answer :—Wherever A is present, there either C is present and B absent, or C is absent. And, inversely, wherever C is present and A is absent, there B is present.

Several other curious combinations might be quoted, still growing out of the equivalence of simple propositions. We are next led to the consideration of Secondary Propositions (hypotheticals, &c.), which the author symbolizes by introducing the idea of *Time* as their peculiarity. A simple, unqualified proposition (affirmative) holds through all time ; a negative, through no time ; a qualified proposition holds only through a certain limited time. The symbol 1 may represent an unqualified truth, as being true through the whole universe of time ; 0 will stand for an unqualified negation, something true for no time. Let X represent a certain proposition, and let x

represent the time of its being true. So, if Y represent another proposition, y may be taken for the time of its being true. Taking both propositions together, $x + y$ will denote the aggregate of the times when both X and Y are respectively true, those times being separated from each other. Again, $x - y$ may denote a remainder of time left when the time y is taken from the time x , it being supposed that x includes y . So, $x = y$ will indicate that X and Y are true for identical times. Further, xy indicates the portion of time when X and Y are both true.

Now, as x denotes the time of X 's being true, $1 - x$ will denote the time that X is false. So $x(1 - y)$ will denote the time when X is true and Y is false: and so on. The same system is to be applied to any number of symbols.

To express the proposition ' X is true' (there being no limit or qualification), we have

$$x = 1.$$

To express the proposition ' X is false—.'

$$x = 0.$$

To express—'Either the proposition X is true or the proposition Y is true (not both).' First, 'When X is true Y is false,' is signified by $x(1 - y)$; 'when Y is true X is false,' is signified by $y(1 - x)$: the equation then is

$$x(1 - y) + y(1 - x) = 1.$$

Next to express the conditional Proposition, 'If the proposition Y is true, the proposition X is true.' This implies that whenever Y is true, X is true; or that the time of the truth of X covers the whole time of the truth of Y , and possibly more. Hence X is at least equal to, if not larger than Y . Consequently some form must be given, implying that Y is contained in X : a form analogous to that required for a universal affirmative proposition. Let v represent an indefinite portion of time, such as to express the unknown part of a whole, 'some, it may be—all,' and the equation required is

$$y = vx.$$

It is unnecessary to exemplify the symbolism for the more complicated cases. The author is so far carried away by the success of his expedient for expressing compound or secondary propositions by a reference to time, that he speculates on an analogous mode of expressing the primary propositions by a reference to space: and thinks that he thus lends some countenance to the doctrine that Space and Time are 'forms of the human understanding.'

A chapter is devoted to the treatment of the secondary pro-

positions, by way of exhausting their whole implication, in the manner previously shewn for the primary propositions; the effect being, however, merely to deduce the usual consequences of disjunctive and of conditional assumptions. It is to be remarked that the process is still one of *immediate inference*, confirming the view that in hypothetical syllogisms so-called, there is no real or mediate inference.

In order to exhibit the value of the symbolical evolution of equivalent forms, Boole selects for analysis two specimens of metaphysical argumentation, sufficiently perplexing to test the powers of a logical method. They are (1) a portion of Samuel Clarke's 'Demonstration of the Being and Attributes of God,' and (2) Spinoza's argument to prove the identity of God and the Universe. He confessed that one main difficulty in dealing with those arguments is to extricate the real premises of the authors; he might have added the farther difficulty of assigning definite and consistent meanings to the very abstract terms made use of by them—necessity, existence, eternity, cause, &c. But the premises once obtained, it is possible to embody them in symbols, and then to extract all their equivalents by solving the corresponding equations. The method may be commended as an interesting effort, varying and corroborating the method followed by a logical and acute mind working upon the *ipsa corpora* of the premises, without symbolism.

We have now reviewed the larger half of Boole's work, and as yet have seen no mention of the syllogism. A short chapter is all that is bestowed upon *mediate inference*; which, however, is a mere carrying out of the algebraic method, with the modifications demanded by the nature of the case.

He begins by accepting De Morgan's additions to the four types of propositions in the common Logic. He lays out the eight forms, with his equations for them: expressing the four new forms by supplying a contrary subject to each of the old forms. The parallelism is shown thus

| | | | |
|-----|--------------------|---------------------|-----|
| A — | All Ys are Xs | $y = v x$ | (1) |
| (A) | All not-Ys are Xs | $1 - y = v x$ | (2) |
| E | No Ys are Xs | $y = v (1 - x)$ | (3) |
| (E) | No not-Ys are Xs | $1 - y = v (1 - x)$ | (4) |
| | = { All Xs are Ys | $x = v y$ | } |
| I | Some Ys are Xs | $v y = v x$ | (5) |
| (I) | Some not-Ys are Xs | $v (1 - y) = v x$ | (6) |

$$\begin{aligned}
 &= \left\{ \begin{array}{ll} \text{Some } Xs \text{ are not } Ys & v x = v(1 - y) \end{array} \right\} \\
 O &\quad \text{Some } Ys \text{ are not } Xs & v y = v(1 - x) & (7) \\
 (O) &\quad \text{Some not-}Ys \text{ are not-}Xs & v(1 - y) = v(1 - x) & (8)
 \end{aligned}$$

The second form of E coincides with A by mere transposition of letters. The second form of I is O, in like manner. The second form of O (O) is the only new form—Some not- Ys , are not- Xs , some things are neither Ys nor Xs . This is one of De Morgan's two disjunctives; his other disjunctive—no not- X is not Y , every thing is either X or Y —does not appear in the above list.

The laws of Conversion follow from the symbolical forms. The proposition 'All Ys are Xs ' being represented by $y = vx$, we have only to read $vx = y$, Some Xs are Ys . To convert the same proposition by negation (obversion and conversion), we deduce, by eliminating v ,

$$y(1 - x) = 0$$

which gives by solution with reference to $1 - x$,

$$1 - x = \frac{0}{y}(1 - y),$$

whose interpretation is 'All not- Xs are not- Ys . [This operation contains methods and symbols not explained in the foregoing abstract].

So far as Conversion goes, the author merely continues his former methods of reducing and interpreting equations; as we might expect from considering that conversion is merely one variety of Immediate or Equivalent Inference. The SYLLOGISM demands a step in advance. The two premises must be embodied in two equations, with a common middle term, and that term must be made to disappear in a third formed out of these two. Thus,

$$\begin{array}{ll}
 \text{All } Xs \text{ are } Ys & x = vy \\
 \text{All } Ys \text{ are } Zs & y = v'z.
 \end{array}$$

Whence, by substituting for y , in the first equation, its value in the second, we have

$$\text{All } Xs \text{ are } Zs \quad x = vv'z.$$

The form $vv'z$ shows that x is a part of a part of z . So with all other cases; it is requisite merely to eliminate the middle term y . The method might be easily carried through the whole of the ordinary syllogisms; as well as applied to the unfigured and fallacious forms. But the author proceeds to deduce the general rules of the syllogism by an equation comprehending all the forms of valid reasoning. He gives as the results of the analysis these rules: 'when one middle term, at

least is universal, equate the extremes.' 'In case of unlike middle terms (one positive and the other negative), with one universal extreme, change the quantity and quality of that extreme, and equate the result to the other extreme : and with two universal middle terms, change the quantity and the quality of either extreme, and equate the result to the other extreme unchanged.'

Suppose the case—

All Ys are Xs

All Zs are Ys.

This belongs to the first rule. 'All Ys' is the universal middle term ; the extremes being equated give as the conclusion,

All Zs are Xs.

Suppose next—

All Xs are Ys

No Zs are Ys.

The proper expression of these premises is—

All Xs are Ys

All Zs are not-Ys.

They belong to the case of unlike middle terms, and have one universal extreme. Whence, by application of the rule, we change the quality and the quantity of that extreme, and equate it with the other extreme—

All Xs are not Zs, or No Xs are Zs.

Commencing from the other universal extreme, we obtain the equivalent result—

No Zs are Xs.

A third case—

All Ys are Xs

All not-Ys are Zs.

Here the terms are of unlike quality. There are two universal middle terms, and, by the rule, we change the quantity and the quality of either extreme (Some Xs into All not-Xs), and equate with the other extreme (Some Zs).

All not-Xs are Zs.

The two last examples are selected by the author as presenting syllogisms that would not be regarded as valid in the Scholastic Logic, which virtually requires that the subject of a proposition should be positive. [As often remarked already, the want of a thorough-going recognition of contraries is the defect of the Aristotelian scheme]. The cases are, however, perfectly legitimate in themselves, and the rules for determining them are undoubtedly *the most general canons of syllogistic*

inference. The analysis employed, the author contends, is not properly of the syllogism, but of a much more general mode of combining prepositions to yield results; and he gives an imaginary case to illustrate this wider import.

Without pursuing the syllogism farther, Boole now discusses the vexed question as to the fundamental type of deductive reasoning, and takes issue with Whately and with Mill, who agree in this that all valid ratiocination is ultimately the inferring of propositions from others of a more general kind; the syllogism being a full and adequate formal representation of the process. Now, as the Syllogism is a species of *elimination*, the question resolves itself into these two determinations, namely, first, whether all elimination is reducible to Syllogism; and, secondly, whether deductive reasoning consists only of elimination.

To the first question, he replies, that it is always theoretically possible so to resolve and to combine propositions that elimination may subsequently be effected by the syllogistic canons, but that the process of *reduction* would, in many cases, be constrained and unnatural, and would involve operations that are not syllogistic.

To the second question, he replies that reasoning cannot, except by arbitrary restriction, be confined to elimination. It cannot be less than the aggregate of the methods founded on the Laws of Thought, and the process of elimination, important as it is, is only one process among others.

He farther remarks that, of all the Laws of Thought, the one of fundamental importance in Logic, is the Law of Contradiction, to which Leibnitz also assigned the same position.

All persons that have attained a just notion of the Relativity of Knowledge, would agree with Boole in the prime importance thus given to Contrariety or Contradiction; but this merely goes the length of Equivalence or Immediate Inference. It prepares the way for Syllogism, and is the main key to the useful enlargements of the syllogism; but it does not touch what is essential to deduction. The axiom, or 'law of thought,' at the foundation of mediate inference must be something else, and if it is not the axiom assigned in the previous chapter of this work, it is an axiom yet to be sought. Passing from Boole's somewhat vague generalities to his actual method, which consists in combining two equations standing for the premises of the syllogism, into a third standing for the conclusion; and adverting to the maxim that justifies the process of reduction,

we seem to see that it is the same maxim as enters into a problem of equations with two or more unknown quantities ; as for example, given $x + y = a$, $x - y = b$, to find x and y . Grant that the conditions of a logical syllogism are fairly expressed by Boole's symbols, and that the algebraic reduction is suitable and relevant to the case, then the logical axiom is the algebraic axiom that permits the substituting for y in one equation, of its equivalent in the other ; as when we obtain from $x - y = b$, $y = x - b$, and insert this value of y in the equation $x + y = a$. The axiom of direct application to the case would be that, for any quantity, its equivalent may be substituted in an equation ; in other words, the substitution, for any quantity, of its equivalent, does not change the value of the equation. This is a various reading of the axiom of mediate equality—things equal to the same thing are equal to one another ; an axiom to which Mr. Mill compares, in point of form, the axiom of the syllogism. If one thing is equal to a second, and the second equal to a third, the first is also equal to the third. In a combination containing A and B, we may introduce in room of B its equivalent C.

A large portion of the work is devoted to Probabilities, in handling which, the author continues the symbolism employed in the previous portion of the work. It is generally admitted that he has made important additions to the theory of this subject, the common ground of Mathematics and of Logic.

CHAPTER III.

FUNCTIONS AND VALUE OF THE SYLLOGISM

1. It is the peculiarity of the Syllogism, that the conclusion does not advance beyond the premises. This circumstance has been viewed in two lights.

On the one hand, it is regarded as the characteristic excellence of the Syllogism.

On the other hand, it is represented as constituting a *petitio principii*.

In the syllogism 'men are mortal, kings are men, kings are mortal,' the conclusion seems already affirmed in the premises.

By virtue of the universal major, coupled with the interpreting minor, there is distinctly involved in the premises the fact that 'kings are mortal.'

(1) To this circumstance has been attributed the peculiar excellence, dignity, and certainty of syllogistic inference. When the two premises are supplied, the conclusion cannot be refused without self-contradiction. There is nothing precarious in the leap from the premises to the conclusion.

The same circumstance has been represented in a more disadvantageous light. The allegation is made that mere repetition is not inference: that to reproduce in a new form what is already given may be highly convenient (as in the various kinds of Immediate Inference), but is no march, no progress from the known to the unknown.

(2) There remains a far more serious charge, and one that takes us direct to the root of Formal Reasoning. Supposing there were any doubt as to the conclusion that kings are mortal, by what right do we proclaim, in the major, that *all men* are mortal, kings included?

It would be requisite, seemingly, to establish the conclusion *before* we can establish the major. In order to say, 'All men are mortal,' we must have found, in some other way, that all kings, and all peoples are mortal. So that the conclusion first contributes its quota to the major premise, and then takes it back again.

This is the deadlock of the syllogism, the circumstance that has brought down upon it the charge of 'reasoning in a circle' (*petitio principii*). In point of fact, we can hardly produce a more glaring case of that fallacy.

The extrication from the puzzle is due to Mr. John Stuart Mill, and the consequence has been a total revolution in Logic.

2. The major premise of a syllogism (in the regular figure) may, so far as the evidence is concerned, be divided into two parts; the one part containing the instances observed, and the other part containing the instances not observed, but inferred.

The major premise, 'All men are mortal,' consists of two very different statements. The first is, that a certain number of men have actually died. The evidence for these is actual observation, the highest of all evidence. The second statement is, that the men now living, and the men yet to be born, will die; for which there is not the evidence of observation.

In the same manner may we analyze any other general

affirmation or negation. The proposition 'transparent bodies bend light' is made up of the bodies that have been actually experimented on, and of bodies that have not been experimented on; in the one case, the predicate is affirmed on the evidence of fact; in the other case, the predicate is affirmed by virtue of the inductive leap from the known to the unknown. Thus, the ordinary form of the general proposition confounds together the observed with the unobserved; the indiscriminate fashion of the two is what has perplexed the theory of the syllogism.

3. In affirming a general proposition, real Inference is exhausted.

When we have said 'All men are mortal,' we have made the greatest possible stretch of inference. We have affirmed mortality of all men, of every class, in every age, past and future. We have incurred the utmost peril of the inductive hazard. Whatever justification needs to be offered for the inference in hand, must be advanced as a security for the major premise.

4. The type of reasoning that best discloses the real process is reasoning from Particulars to Particulars.

The basis of fact in every argument may be stated to be the particulars actually known from experience; as the mortality of the men that have died. The inference is usually to some other particulars unobserved, as 'the present inhabitants of London will die.' The real evidence for the mortality of the men now living is the death of their predecessors. A, B, and C, have died; D, now living, will die.

The practice of reasoning at once from certain particulars experienced, to some other particular as yet unexperienced, (there being a similarity in the cases) is not only the usual, but the most obvious and ready method. We feel that the real force of every reasoning lies not in the general statement, but in the actual facts; and we are as much moved by the facts in their particularity, as when they are given in a generality. That boiling water will scald the hand, is sufficiently proved by its having done so in innumerable past instances; the deterring force lies in these actual instances. We are influenced by individual precedents, as strongly as by rules.

This is seen extensively in all professions. The experience of a professional man consists of the cases he has actually ob-

served ; these he remembers as particulars, and when a new example is presented, he at once assimilates that with the previous particulars, and infers accordingly. When Dr. Mead was called in to the last illness of Queen Mary, he pronounced the disease to be small pox ; his knowledge of that ailment, was the remembrance of a series of patients previously witnessed by him ; the queen's symptoms *resembled* those, and he drew the inference.

5. Wherever we may infer from a certain number of particulars given, to one other particular, we may infer to a whole class, or make the inference general.

If we can infer, from the men that have died, that the present Pope will die, it is by virtue of a sufficient amount of resemblance between them and him ; and we must be prepared to make the same inference in all other cases where the resemblance holds. We may, therefore, say once for all, whoever resembles past generations of human beings, in the points wherein the pope resembles them, will die. The justification of one is the justification of the whole. The inference to an individual case must not be arbitrary ; it must be grounded on a resemblance, and be applicable wherever the resemblance is found.

In a general proposition, therefore, we state the *points of resemblance* that entitle us to infer from past particulars to a new particular ; and in stating these points we render the inference at once general, and *formally* exhaustive. We mingle up in one statement the observed known, and the inferred unknown, the evidence and the conclusions. The use of general language enables us thus to rise beyond particular inferences.

6. Deductive Inference may be described as a process of Interpretation.

Although the major premise covers the conclusion, it does not point to it by name, but only by character. The premise 'men are mortal' does not specify kings, nor the living pope ; it indicates certain marks by which we are to judge whether kings and popes are to be pronounced mortal, namely, the marks of 'men or humanity.' Something, therefore, is wanting in addition to the major premise, in order to the conclusion, the pope is mortal ; we have to be assured that he is a man, that he conforms to the defining marks of human beings. To supply this requisite is the purpose of the minor premise,

which declares that the pope possesses the attributes of men, or *identifies him* with the *subject* of the major premise. The necessity for such an affirmation rescues the syllogism from Immediate Inference or tautology. 'All men are mortal' includes 'the pope is mortal,' on the supposition that the pope is a man; and if this supposition is explicitly given in a distinct proposition, the pope is then brought within the sweep of the major premise: and the conclusion is established.

.After affirming a general proposition (or making a general denial) connecting or disconnecting a certain subject with a certain predicate—men and mortality—we have still to hunt out the particular cases of the subject, the things that possess its attributes. This is the *real deduction*, and it is a *material* and not a formal process. It is an operation of comparing the actual individuals already pointed out by the generalized subject—actual and known men—with all future individuals as they occur, and of pronouncing agreement of the new with the old. The deductive inference that 'the pope is mortal,' presupposes an examination (direct or indirect) of the pope's personality. If this resembles the usual type of humanity, judged from the instances actually known to us, we identify him with the subject, 'men,' in our general proposition. The identity being considered satisfactory, we complete the syllogistic formula, and declare him to be mortal.

The proposition 'men are mortal,' by its form of universality, imposes upon us, and leads us to suppose that we have in our grasp the whole human race. The correcter view is to regard it as an allegation respecting a certain number, with a power of including others as they come on the stage. The proposition assigns marks for the future identification of the beings that are to be declared mortal; and, as the identification proceeds, the minor premise is replenished with appropriate cases, and so brings about the conclusion.

The interpretation of a law or a command illustrates the purely deductive part of the operation of reasoning—the supplying of the minor. The law is given in general terms; certain characters are assigned as belonging to the subject of the proposition. The administrator or judge ascertains whether any particular case has or has not the characters specified. If it has, a minor proposition is afforded, and a conclusion is drawn.

This case also shows that the syllogism is the mere formal completing of an operation, not at all formal, but in the strict sense material. The operation consists in comparing one par-

ticular fact with other particular facts, through the medium of a general description. The wording of a law, however general be the terms, must be such as to suggest definite individual cases. When the law mentions heritable property, or personality, it must either state or suggest the particular things intended; and the question of the application to a given case turns upon the comparison of the case with the cases cited or suggested by the general term or definition. Hence, the business of the reasoner, in actual practice, is *concrete comparison*, from which, in the last resort, he can never be exempted. This is *material deduction*, which, in its essence, is the same as *material induction*, being the carrying out of the inductive operation, or the in-gathering of the details shadowed forth, but not actually seen, in the general proposition.

Legal decisions are founded sometimes on statutes, sometimes on precedents or previous decisions. There is no generic distinction between the two modes. A statute has no meaning except the particular cases specified or suggested; and a precedent must involve a principle or rule. In both, the judge refers back to concrete particulars, which are viewed under a certain point of likeness or community.

Another case is the application of general theorems furnished by the observations of others, such as the principles of science established by foregone researches. We may have had no share in arriving at the induction known as the atomic theory; we have not even seen the facts, we receive them embodied and *registered* in the general statement of the law. We must understand the meaning of that statement; we must realize the kind of facts intended by it. When a case is started, a given compound of two substances, we must say, by concrete comparison, whether this compound has the characters of the compounds expressed as chemical compounds. For example, is the atmosphere a chemical compound? Does it agree with the general characters of chemical compounds, or with those *typical instances* that the general characters can do nothing but refer us to. This is a truly material deduction; it is that process of comparing instances that is the essence of the generalizing operation, as seen in induction. It exactly resembles generalization with a view to definition.

7. Although the deductive stage of induction is still an inference from particulars to particulars, which nothing can supersede, there are certain advantages in embodying the possible inferences in a formal generality.

Mr. Mill remarks that the syllogistic form of inference, from generals to particulars, which supposes that each induction is made general, is 'a collateral security for the correctness of the generalization itself.' It is so in two ways.

First. It increases the sense of responsibility on the part of the reasoner, by letting him know that his inference to one individual must equally apply to a large host of individuals. A common device for checking a rash inference is to point out the extent of the consequences involved. The legal decision against John Hampden, in the matter of thirty shillings of ship money, was portentous as affirming the king's power to tax the nation without a parliament.

Secondly. If an induction is unsound, the making it general is likely to suggest contradictory instances. This is merely a modification of the same consequence. Any person attempting to justify a particular despotism must be prepared to say that, in all similar circumstances, despotism would be desirable. The remark is sometimes made, in the controversy as to the inspiration of the Bible, that even Milton was inspired; but, if so, then all great poets—Homer, Virgil, Dante, Chaucer, Shakespeare, Dryden, Byron, Shelley—must also own the gift of inspiration.

Mr. Grote, in defending the received canon of the Platonic writings from the critics that would reject many of the Dialogues, on the ground of their style being unworthy of Plato, points out the numerous Dialogues that would have to be sacrificed to this criterion, if each critic were allowed to reject for himself, and all rejections were admitted.

8. One great use of the syllogistic form is to analyze, bring to light, and present for separate consideration, the parts of a step or a chain of reasoning.

This has been already exemplified in the applications of the syllogism to confused reasonings. It is advantageous to know that the truth of a conclusion by inference supposes the truth of two separate allegations, both alike necessary to the conclusion. To prove that A is C, by a mediate inference (B is C, A is B), two propositions have to be verified; and the mind is aided in disentangling a perplexed argumentation, by knowing what to look out for.

In stating the distinction between the two modes of reasoning, used both in Law and in Politics—reasoning from Precedents or Examples, and reasoning from Rules or Principles—Sir G. C. Lewis adverts to the great superiority of the last, the reasoning

from Rules. The reason of the comparative obscurity of the argument from example or precedent, is that the principle involved is usually suppressed. 'The reasoning is much more perspicuous when the general principle is stated first, the particular case is placed under it, and the conclusion is then drawn. In order to argue from one case to another, it is necessary to reject from each the circumstances immaterial to the matter in hand, and to compare those in which they agree. In complex cases, this process is often extremely difficult. Much sagacity and knowledge of the subject are required, in order to discriminate between material and immaterial facts—to reject enough, but not more than enough. For if immaterial facts are retained, the comparison becomes obscure and uncertain; if material facts are rejected, it becomes fallacious. This process, which, in the argument from precedent, must often be performed mentally, though it may be easy and sure to the experienced practitioner, perplexes the tiro. Hence, students of the law have great difficulty in collecting legal rules from cases, though they are soon able to apply a rule of law, laid down in general terms, to a particular case of practice.'

CHAPTER IV.

TRAINS OF REASONING AND DEDUCTIVE SCIENCES.

1. A series of syllogisms may be connected in a chain.

Logicians have always recognized compound reasonings. The *Sorites* is a connected chain of syllogisms. The conclusion of one syllogism may be the major premise to a second, and so on.

The *Sorites* is usually stated in this form:—

A is B, B is C, C is D, &c., therefore A is D.

The regular form of proof (by the First Figure of the Syllogism) is—

B is C, A is B, therefore A is C.

C is D, A is C, therefore A is D, &c.

It can scarcely ever happen that a proper deduction in this simple form can be protracted over two or three syllogisms. The application of a universal proposition to a particular case seldom needs to descend by three or more distinct steps: indeed, in by far the greater number of instances, the descent is made at once.

No new logical principle, or modification of principle, is involved in these consecutive reasonings. Their lucid state-

ment is a matter of consideration for the expositor, but they present no speciality to the logician. Still, they are usually discussed in treatises on logic; and we may, following the example of Mr. Mill, take occasion from them to discuss two themes—the compatibility of the foregoing theory of the syllogism with such trains, and the nature of the Deductive Sciences.

2. A chain of Reasoning is reducible to a series of syllogisms, the major in each being an induction from particulars, or a truth ultimately based on particulars.

Thus, if we were to prove that intelligent beings, although they may be interrogated, are not to be experimented on like brute matter, we should have the following chain:—wherever there is intelligence, there is sensibility, in other words, susceptibility to pleasure and pain; we are not at liberty to inflict pain; now, most experiments that could be tried upon sentient creatures would be painful; hence, intelligent beings are not fit subjects for experimental enquiry. Three syllogisms are concerned in this chain of reasoning. The majors are—

- (1) Society prohibits the infliction of pain.
- (2) All intelligent beings have sensibility to pain.
- (3) Experiments for ascertaining function in sentient beings lead to pain.

Each of these majors may be resolved, according to the method of the previous chapter, into particulars observed and particulars inferred, or left to be inferred, by virtue of identity. The first major (Society prohibits) is in the form of a command, the case where we may be supposed to be least concerned with the particulars, and most concerned with the general description serving to identify the particulars. Still it must not be forgotten that the real force even of a command is embodied in the instances where it is enforced; the general statement means nothing, is nothing, except as referring us to these; the application of the rule is an inductive extension of these instances. The second major (intelligent beings have sensibility) takes in the observed coincidences of intelligence and sensibility, together with the future extensions of these by identification with the presence of intelligence—the first term of the couple. The third major is likewise an inductive generalization, containing the observed particulars where experimenting has ended in pain, together with the resembling inferred particulars.

We may arrange the train of reasoning in syllogisms. Thus, —taking a different order—

First Syllogism.

Experiments for ascertaining function in sentient creatures lead to pain.

The present proposal is an experiment for ascertaining function.

The present proposal will lead to pain (*Barbara*).

Second Syllogism.

Society prohibits the infliction of pain.

The present proposal will lead to pain.

Society prohibits the proposal to experiment on sentient beings (*Cesare*).

Third Syllogism.

Society prohibits experiments on sentient beings.

All intelligent beings are sentient beings.

Society prohibits experiments on intelligent beings. (*Cesare*).

The form (Society prohibits, &c.), has the force of a negative; were it not so, the last syllogism would not be valid.

The language of inference from particulars to particulars might be used in each of these syllogisms. Thus in the first: Experiments for ascertaining function in sensitive beings have been observed to lead to pain; the present case is an experiment for ascertaining function: the present case will lead to pain (as the observed cases have done). Similarly for the others.

The Deductive Sciences

3. The Deductive Sciences are those where the labour mainly lies in applying or carrying out ascertained inductions, that is, in the discovery of minors to given majors.

From the foregoing theory of the syllogism, it is apparent that every deduction supposes a previous induction. The Deductive Sciences, therefore, do not dispense with induction. Whereas, in the Inductive Sciences, such as Chemistry and Physiology, the chief labour consists in arriving at inductions; in the Deductive Sciences, as Mathematics, the inductions are few and easily gained (being in fact sometimes called intuitions) and the labour consists in carrying them out into their various applications, by bringing cases under them. We soon arrive at the inductions 'things equal to the same thing are equal,' or 'the sums of equals are equal:' 'the differences of

equals are equal : ' but it was not easy to bring under the sweep of these inductions the proposition 'a sphere is equal to two-thirds of the circumscribed cylinder.' This is arrived at only after a long and circuitous process of successive deductions, based upon the invention of numerous diagrams.

If we take a comparatively simple case of geometric deduction, the 47th of the First Book of Euclid, 'the square described on the hypotenuse of a right-angled triangle is equal to the sum of the squares described on the two sides,' we shall find that the proof can be accomplished by two main leaps—two syllogisms having axiomatic majors, and a preparatory syllogism having as its major a previously established derivative proposition. The rest of the process is not syllogistic. We first, by an ingeniously devised construction, establish two minors under the proposition—'A parallelogram and a triangle being on the same base and between the same parallels, the parallelogram is double of the triangle ;' and then proceed to the main steps, the application of the axioms. We first apply the axiom—'The doubles of equals are equal,' (a derivative from the axiom—'The sums of equals are equal,') to prove that the square described on one of the sides is equal to a part of the hypotenuse square, and that the square described on the other side is equal to the remaining part of the hypotenuse square. This being done, it needs but an easy application of the axiom—'The sums of equals are equal,' to complete the proof.

The deductive sciences circumvent their problems ; they accomplish indirectly what there is no means of accomplishing directly. The science of mathematics instead of resting satisfied with announcing its axioms and definitions, and leaving people to apply them at once, evolves a vast scheme of deductive properties, to any one of which we may repair in an emergency, instead of making a connexion at once with the fountain head. We measure a height by bringing the case under some theorem of Plane Trigonometry that chances to be adapted to the means at our command.'

The length and the complicity of mathematical or other reasonings may be ascribed to these two circumstances.

(1) There are many steps of mere Immediate Inference, as in applying Definitions. Thus, when Euclid shows that two figures coincide, he makes a formal appeal to the Definition of Equality (namely, Coincidence), and, by virtue of that declares them to be equal. This is seemingly a step in the reasoning ; it involves a distinct act of attention on the part of the stu-

dent, but it is not a deduction or syllogism. So, there may be steps involving other transitions to Equivalent Forms, as Obversion, Conversion, &c.

(2) Not only is a great deal of preparatory construction or scaffolding often required in order to bring the case under the sweep of a previous generality, but, when the construction is made, there jut out from every part of it separate inferences, and all these have to be made convergent to the purpose in hand. Moreover, many propositions start at once with a complicated hypothesis—‘If a point be taken without a circle (1), and straight lines be drawn from it to the circumference (2), whereof one passes through the centre (3),’ &c.; the proof in these cases is a convergent series of steps, each starting from a distinct member of the hypothesis.

The process of Identification to supply a minor is difficult according to the complicity of the subject of the major; as in Diseases, in Law, in Politics, &c. A disease being characterized by three, four, or five distinctive symptoms, must be identified on all these symptoms; a failure in any one leaves the disease unidentified. Hence, deduction may be a work of labour even in the sciences of Induction, as Medicine must be pronounced to be.

So, in Politics, Sir G. C. Lewis remarks that the difficulty may lie in bringing the Premises of the syllogism together, that is, in finding the major to a given minor, or the minor to a given major. ‘It is the subsumption of the minor under the major premise that really constitutes the originality, or invention, of the argument.’ The following is an example:—

General Maxim, or Major—When a customs duty is so high as to produce extensive smuggling, it ought to be reduced.

Particular case, or Minor—The existing customs duty, in country A, upon tobacco, or brandy, or hardware, &c., leads to extensive smuggling.

Now, the minor is obviously a matter of fact (determined partly by reasonings from facts), and may take much trouble to establish.

4. The special aim of Deduction is to ascertain every fact implied in facts already known. A Deductive determination is opposed to an Experimental determination.

When, by the application of ascertained inductions, we can discover new truths, we save the appeal to direct experiment. By the parallelogram of forces, we can find the exact course of any moving body urged in different directions by given

forces.' A process of computation is substituted for a process of observation; the consequence is, in most instances, a great economy.

The pushing of truths of induction to all their deductive applications is one great department of scientific research. The aptitude for the operation is almost purely intellectual. When a great law, such as Gravitation, has been established, the following out of all its deductive consequences supplies work to several generations of men. The generalization of the present day, called the Persistence of Force, will give probably an equal amount of occupation to the more purely deductive or speculative aptitudes of the scientific mind. The inductive laws that connect Mind with Body, when ascertained with precision, will admit of being deductively pushed in numerous ways, and will yield many facts at present discoverable only by separate observations. The doctrine of the Relativity of all Feeling and Thought has not as yet been completely followed out to its consequences.

CHAPTER V.

DEMONSTRATION.—AXIOMS.—NECESSARY TRUTH.

1. The kind of evidence named 'Demonstration' has its sources in Induction.

Demonstrative proof is only another name for Deductive proof, which, in the last resort, is Induction. The propositions of Euclid are said to be demonstrated; and, as above seen, this means that the conclusions are proved by bringing each case under the sweep of the fundamental principles of the science.

To make out Mathematical Demonstration inductive, it is requisite to show—(1) that the foundations of the Science (the axioms) are inductive; and (2) that the axiom of the Syllogism is inductive. The axioms of mathematics supply the principles, and the axiom of the syllogism justifies their application.

In the question respecting the ultimate foundations of the so-called axioms, these are the chief examples in dispute. It is maintained, on one side, that the axioms of Mathematics,

the axiom of the Syllogism, together with the axiom of 'Causation,'—are inductions from particular facts of experience; and on the other side, that they are of intuitive origin, and, in this origin, possess a higher certainty than can be given by experience.*

2. The chief argument against the Inductive origin of these principles is that they are *necessary*, and no experience can give the character of necessity.

The idea of 'necessity,' as attaching to such truths as the mathematical axioms, dates from Leibnitz; it was re-stated, in a qualified form, by Kant, and persists in the minds of many to the present day. The term, however, is ambiguous.

Meanings of Necessity.

3. I. In common speech, 'necessity' is a synonym of certainty; and would apply to *inductive* truths.

When speaking of anything that is certain to happen, we use among other words, the term 'necessary.' We should call the freezing of water, at 32°, a necessity, meaning that we are perfectly sure of its happening. We even say that vice is a necessary consequence of bad training.

The necessity in such cases has admittedly nothing to do with intuitive perception. Experience is competent, in every instance, to give the strong assurance that the word signifies. So, we have only experience to rely upon in believing that the sun must rise to-morrow.

There could be nothing incompatible with this usage in terming all the inductive laws of nature 'necessary'—the law of gravity, the laws of motion, the fundamental laws of organization, and so on. But metaphysicians are accustomed to call these principles 'contingent,' as opposed to necessary; for although they are true, as the universe is now constituted, they might have been otherwise. The law of gravity might have been wanting; the laws of organized beings might have been different. But, in no circumstance (it is said) could 'two straight lines enclose a space;' this, therefore, is necessary in a more peculiar sense of the word, as will be next stated.

* On the subject of Mathematical Evidence, other questions have been raised, namely, the place of the Definitions in the Science, and the supposed hypothetical character of definitions. These questions will be adverted to afterwards (LOGIC OF THE SCIENCES, *Mathematics*).

4. II. 'Necessity' more properly means *implication*; 'necessary truths' in this sense are the truths demanded by Consistency. Their denial is a contradiction in terms.

These truths have already been fully exemplified. (See INTRODUCTION, and also EQUIVALENT PROPOSITIONAL FORMS). That the less cannot contain the greater, is necessary; it follows from the very meaning of less and greater; it could not be contradicted without declaring the greater not to be the greater. 'The same thing cannot be in two places at once' is necessary; the meaning of a 'place' is some definite spot the negative of all other places; to say that a thing is in a particular place is to deny that it is in a second, or a third, or any other place. 'Time is an eternal *now*!' must be set down as self-contradictory.

Some of the axioms of Euclid are necessary in this sense. 'A whole is greater than its part' is implicated in the definition of whole and part; it could not be contradicted without contradicting the definition. A whole is summed up by its parts; omit any of these, and the whole is not made up; the result is something less than the whole.

'Things that coincide are equal' is not an axiom but a definition; it is the mark or test of equality, the only mark that can be propounded in the last resort.

Of all the alleged necessary truths, the one most frequently cited in the present controversy is—'Two straight lines cannot enclose a space.' This was held by Kant to be a *real* proposition, a *synthetic* judgment; in other words, the subject is not implied in the predicate; to it the criterion of 'implication' would, therefore, not apply.

On the other hand, mathematicians are now probably unanimous in regarding this as a corollary from the definition of the 'straight' line, or as implicated in the very essence of straightness; so that to deny it would be a contradiction in terms. They would characterize it, in Kant's own language, as an 'analytic' judgment. A very little reflection on the case proves that the mathematicians are right. Starting from the definition of the straight line—'when two lines are such that they cannot coincide in two points without coinciding altogether, they are called straight lines,' we see that the very terms forbid the enclosing of a space; what meaning can we attach to 'coinciding altogether,' but the exclusion of non-coincidence, or of an intermediate space? Total coincidence, and an intervening space, are wholly incompatible; if the one-

is true the other is false. The proposition is therefore necessary in the sense of implication, as much so as a 'straight line is not a bent line,' 'a whole is greater than its part.'

The axiom 'Things equal to the same thing are equal to one another' is not a truth of implication, and therefore is not a necessary truth in the present sense. The subject and the predicate express distinct properties, and the one does not involve the other. The axiom declares that *mediate* coincidence is to be held as carrying with it, or as making, *immediate* coincidence; but the two modes of coincidence are not identical. It is *immediate* coincidence that makes equality, according to the *definition* of equality; the axiom extends this very narrow, and often inapplicable test, and declares that coincidence *through some third thing*, a go-between, will be found in the end to be the same as actual coincidence, and is consequently to be accepted in all cases as a test of equality. If, therefore, this axiom is to be held as a necessary truth, some other meaning than the present must be assigned to necessity.

5. Necessary truths, in the foregoing signification, are so far independent of experience, that they are perceived to be true when the language is understood. They do not, however, require any powers of intuitive perception.

As soon as we fully comprehend the notion of whole and part, we perceive that the whole is greater than the part; we do not need to make observations and experiments to prove it. We required concrete experience, in the first instance, to attain to the notion of whole and part; but the notion once arrived at implies that the whole is greater. In fact, we could not have the notion without an experience tantamount to this conclusion. When we know a fact, we know it, even when called by another name, which is all that is meant, at present, by necessary truth. When we have mastered the notion of straightness, we have also mastered that aspect of it expressed by the affirmation, 'two straight lines cannot enclose a space.'

No intuitive or innate powers or perceptions are needed for such cases. Our ordinary intellectual powers enable us to pronounce, in more than one form, that an object is everything or anything that we have found it to be. We cannot have the full meaning of 'straightness' without going through a comparison of straight objects among themselves, and with their opposites, bent or crooked objects. The result of this comparison is, *inter alia*, that straightness in two lines is seen to

be incompatible with enclosing a space; the enclosure of space involves crookedness in at least one of the lines.

6. III. A third meaning and criterion of Necessity, is *inconceivability of the opposite*.

It is maintained that 'things equal to the same thing are equal to one another,' because the mind is unable to conceive things agreeing with a common standard, and yet not agreeing when directly compared. It is also maintained that we are unable to conceive 'effects arising without a cause;' whence such propositions are declared to be true necessarily. The test of inconceivability of the opposite (strongly urged by Whewell, and held with modifications by Spencer), is liable to serious objections. What we can, or cannot conceive, is manifestly dependent, in a very large measure, on our education: the proof of which is that many truths inconceivable in one age and country are not only conceivable under a different state of education, but are so thoroughly engrained that their opposites are inconceivable. The Greeks held matter to be eternal and self-existent; many moderns hold that the self-existence of matter is inconceivable. Some maintain that mind is the only conceivable source of moving power or force; others, regarding the action of mind upon matter as inconceivable, have contrived special hypotheses to get over the difficulty,—we may instance Malebranche's doctrine of Divine Interference, and Leibnitz's Pre-established Harmony. Newton could not conceive gravity without a medium.

With regard to truths of Implication, the difficulty of conceiving the opposite must be at its maximum. Yet self-contradiction is not an impossible operation, for it is often done. In Theology, people have even boasted of holding contradictory propositions. But where the subject does not imply the predicate, there is no self-contradiction, and the opposite of any such proposition may be conceived. That things mediately coinciding, should not immediately coincide, is conceivable; for the facts are different; the difficulty that we feel is in contradicting our habitual experience on a matter so very familiar and tangible.

Propositions of avowedly inductive origin may be so strongly associated that their opposites are all but impossible to conceive. It is scarcely in our power to conceive colour without extension; and yet the two are united solely by our experience; they strike the mind through different avenues, and their incessant conjunction constitutes a practically indissoluble

bond. We should have some difficulty in conceiving soot flakes, particles of dust, and small pieces of paper, falling to the ground plumb and swift like a stone. The Greek proverb for the impossible was water flowing back to its source.

The Nature of Axioms.

7. The fundamental principles of the Deductive Sciences are called Axioms.

Every Deductive Science must begin with certain fundamental assumptions. In Mathematics, and in Logic, these are deemed so self-evident, that no express effort is made to establish them. In Mechanics, the statement of the Laws of Motion is accompanied with a few examples to make them at once intelligible and evident. In Chemistry, the Atomic Theory is somewhat too far removed from ordinary comprehension to be called a self-evident axiom, albeit the most fundamental assumption contained in the science.

The requisites of an axiom are, first, that it should be a *real* proposition, and not a definition; and, secondly, that it should be *independent* of any other principle within the science.

On the first of these two requirements, we should have to reject Euclid's axioms—'Magnitudes that coincide are equal,' and 'The whole is greater than its part.'

On the second requirement, we must reject,—

The differences of equals are equal;

If equals be added to unequals, the wholes are unequal;

If equals be taken from unequals, the remainders are unequal;

Doubles of equals or of the same are equal;

Halves of equals or of the same are equal;

Two straight lines cannot be drawn through the same point, and parallel to the same straight line, without coinciding.

It may be useful to give an explicit statement of these truths, but as they are all derivable from other axioms (together with Definitions), they should be appended to these others, as corollaries or inferences. If, in any instance, we set up a *derivative* proposition as an axiom, we break down the sole boundary between axioms and the propositions or theorems constituting the body of a science.

8. The only two Axioms of Mathematics, properly so called, are, the axiom of 'mediate coincidence,' and the axiom of the 'equality of the sums of equals.' These are Inductive truths.

The excision of Definitions with their corollaries, and of Derivative Propositions, leaves only the two axioms now mentioned—‘Things equal to the same thing are equal,’ and ‘The sums of equals are equal.’ These are real, and not essential or analytic, propositions: and they are ultimate within the science. They are two distinct tests of equality, over and above the defining test, immediate coincidence. From them, together with the definition, all other tests of equality are deducible.

To say that they are Inductive truths, generalizations from our experience of the particular facts, is to say that they have the same origin as the great mass of our knowledge (not deductive). That day and night alternate, that water flows downward, that smoke ascends, that plants grow from seed, that animals die, that men seek pleasure and eschew pain,—are all obtained by a comparison of observed facts; and this is the regular, the usual source of scientific generalities. The burden of proof lies upon those that would assign any other source to the two axioms named; some reasons must be given to show that they are exceptions to the prevailing rule.

The chief reasons actually assigned are those already examined, their Necessity, and the Inconceivability of their Opposites. As corroborating these, or rather as putting in a different shape the supposed difficulty of referring the axioms to experience, it is said that the intensity of *our conviction* that ‘things equal to the same thing are equal’ *is greater than could arise from the accumulated comparisons that we have instituted on actual things.* The considerations that serve to obviate what force there is in this objection are the following.

First, by the law of Belief already explained, every *uncontradicted* experience has, on its side, all the force of our primitive credulity. The initial believing impetus of the mind errs on the side of excess; and if nothing has happened to check it in a particular case, it will be found strong enough for anything.

Secondly, our opportunities of comparing magnitudes are numerous and incessant; they require only the very simplest and most accessible instruments. The child, having at command, three equal chips of wood, cannot avoid making, in the course of an hour, scores of comparisons that exemplify the axiom of mediate equality.

Thirdly, it is usual to remark, on the mathematical axioms generally, that the subjects of them—namely, magnitudes and forms—are with the greatest possible ease represented in ima-

gination, so that we can make numerous ideal experiments, in addition to our comparison of actual things in the concrete.

9. The Axioms of the Syllogism repose upon experience.

In the form—'Attributes co-existing with the same attribute, co-exist,' we have a principle closely resembling Euclid's first axiom of Equality; the character of the evidence for both must be the same. Now, so far is this axiom from being 'an absolute and intuitive certainty, that it is erroneous. We may illustrate it by a parallel form, 'Things in contact with the same thing are in contact with one another;' which is plausible but fallacious.

The *dictum de omni et nullo* cannot be exempted from the criterion of experience. It is not intelligible without much familiarity with examples of the generalizing process; and, as, in the case of all other first principles, the same knowledge that makes it understood, suffices to verify it.

However expressed, the Axioms of the Syllogism are, in the first place, Real Propositions, and not identical statements under the so-called Law of Identity, or Self-Consistency. And, in the second place, as Real Propositions, they are not intuitively suggested to the mind; they grow up with our experience, and if our belief in them seems to outrun experience, the same thing happens to all our beliefs.

10. As regards the Law of Causation, usually included among the so-called *a priori* elements of our knowledge, there is a strong primitive tendency to believe it in a crude form, while experience must adapt this belief to the actual facts.

We have already seen that the primitive tendency of the mind is to believe, until checked, that what is now will continue, that what is here is the same everywhere. Neither experience nor any intellectual faculty creates this impetus; but experience arrests and modifies it, till by degrees it adapts itself to the real occurrences. The headlong impulse is curbed in such matters as the surrounding temperature, luminosity, and visible appearances; it is left in possession of other matters, as the force of gravity. The instinct is important as giving the active element of belief; it is perfectly worthless as a guide to the things proper to be believed. So far as concerns the authority or evidence, for causation, experience is paramount over instinct; apart from experience, the infant would for life believe that all the water of the globe is of the temperature of its first bath.

The crude impulse to believe that what is will continue, after the shock of many contradictions, is transformed into a belief in the uniformity of nature, as represented by the law of Causation.

- 11. The axiom underlying the axioms of Mathematics, and the axiom of the syllogism, is the axiom of the Uniformity of Nature.

• The consideration of cause and effect brings us face to face with the most fundamental assumption of all human knowledge, expressed by such language as 'Nature is Uniform' 'the Future will resemble the Past', 'Nature has fixed Laws.' This axiom is the common ground of all inference, whether avowedly inductive, or induction disguised under the forms of deduction. Without this assumption, experience can prove nothing. We may have found, in ten thousand instances, that magnitudes coinciding with the same magnitude also coincide when applied to one another; so far as these instances go, the fact is not to be disputed; the evidence of actual trial is the highest we have. But they do not prove that it will happen in any untried instance. This must be received without proof; it can repose on nothing more fundamental than itself. If we seem to offer any proof for it, we merely beg it in another shape. (See APPENDIX D.)

APPENDIX.

A.—CLASSIFICATION OF THE SCIENCES.

It is here proposed to subjoin a short account of the different modes of classifying Science or Knowledge. The subject has various logical bearings. The concatenation of Knowledge is in itself a Logic.

The mode of partitioning Knowledge that first gained attention was Bacon's threefold division into HISTORY, PHILOSOPHY, and POETRY; in correspondence with the three great modes of intellectual production, or faculties—*Memory, Reason, and Imagination*. HISTORY, the product of Memory, deals with *individual* things; PHILOSOPHY, the product of Reason, compares, classifies, and works up these materials; POETRY, the product of Imagination, is the department of fiction, fable, or creation, as opposed to the literal rendering of things in History and in Philosophy.

In dividing and sub-dividing these leading departments, Bacon displays his usual copiousness. HISTORY is divided into *Natural History* and *Civil History*. *Natural History* is the collective matters of fact of the world, laid out under Celestial Bodies, Meteors, the Earth, &c. *Civil History* is Ecclesiastical, Literary, Political, with minor sub-divisions.

PHILOSOPHY refers to God, to Nature, and to Man. The first head gives Theology. The second is a somewhat crude syllabus of Mathematics, Natural Philosophy, and Metaphysics. The Philosophy of Man is divided and sub-divided in much curious detail, but with no logical precision. He speaks of man in a three-fold aspect—(1) Man in general, (2) the human body, and (3) the human mind. The theoretical and the practical aspects of our knowledge respecting humanity are indiscriminately mixed.

As a first attempt at partitioning the totality of Literature, the scheme of Bacon deserves to be commended. But the lines of demarcation are for the most part vague and unsatisfactory. The distinction of Individual (as History) and General (as Philosophy) is wholly unsuited to a primary division

of knowledge; we cannot divorce the particulars from the generalities in the same subject matter.

The main outline, as regards the three-fold Division, was maintained in the classification of D'Alembert, intended for the plan of the French 'Encyclopédie'; but with great improvement in the sub-divisions. The sub-division of Philosophy, relating to *Nature*, is a methodical arrangement of the Mathematical, the Physical, and the Biological Sciences, together with the more Scientific Arts, as Medicine, Agriculture, and Metallurgy.

The Natural History department of HISTORY includes Meteors, Geography, Minerals, Plants, and Animals, very much on the scheme of Bacon, with the curious detached addition (also after Bacon) of a division for Prodigies, or deviations from the usual course of Nature.

The Science of Man is distributed under the two heads Logic and Morals. *Logic* comprises the arts of Thinking, Retention, or Memory, and Communication. *Morals* is General, that is, regards Virtue at large (Ethics); or Particular,—including Law or Jurisprudence. This is the mode of approaching the science of mind that has been embodied in our Universities. Excepting in recently founded schools, there is no chair for Psychology or the Theoretical Science of Mind; the subject is left to come under Logic and Moral Philosophy; the Intellectual Powers being described in the Logic course, the Active Powers in Moral Philosophy.

Thus, in D'Alembert, as well as in Bacon, there is total confusion of the Theoretical and the Practical.

The plan of subjects in the 'Encyclopedia Metropolitana,' (began to be published in 1815), is worthy of being quoted. There are four Divisions in the work.

The First Division includes PURE SCIENCES, divided into FORMAL—Grammar, Logic, Rhetoric, Mathematics, Metaphysics; and REAL, Law, Morals, and Theology.

The Second Division is the MIXED SCIENCES,—Mechanics, Hydrostatics, Pneumatics, Optics, Astronomy [constituting the larger part of our usual course of Natural Philosophy].

The Third Division is the APPLIED SCIENCES, subdivided into EXPERIMENTAL PHILOSOPHY—Magnetism, Electricity, Heat, Light, Chemistry, Acoustics, Meteorology, Geodesy; FINE ARTS; USEFUL ARTS; NATURAL HISTORY (with applications to Medicine).

These are the properly scientific divisions; the other sub-

jects are History, Biography, Geography, Lexicography, and Miscellaneous information.

The designations 'Pure,' 'Mixed,' and 'Applied' Sciences have characteristic meanings, although not precisely carried out in the above scheme. The Pure Sciences are the 'more Abstract and Formal Sciences, not involving the consideration of objects in the concrete; the two leading examples are Mathematics and Formal Logic. The Mixed Sciences consider the applications of the laws of the Formal Sciences to actual things. The Applied Sciences, in so far as distinct from the Mixed Sciences, should be equivalent to the Practical Sciences.

Dr. Neil Arnott, in his work on 'Physics,' published in 1828, gave wide publicity to a division more in harmony with our present views. He distributed the leading sciences under four heads, representing the four classes of general Laws of Nature—namely, *Physics, Chemistry, Life, and Mind*. He viewed *Mathematics* as preliminary and indispensable to these, being the Science of *Quantity*, or Measure, but not a department of natural operations, in the same acceptation as *Physics* or *Chemistry*. All the sciences give foundation to Arts.

In his subsequent treatise, entitled 'Survey of Human Progress,' Dr. Arnott brought out more decisively the distinction between Sciences and Arts, and between the Concrete and the Abstract Departments of Science. Concrete Science he calls the knowledge of THINGS; and he enumerates, under this head, Astronomy, Geography, Mineralogy, Geology, Botany, Zoology, the History of Man. Science, or Philosophy (Abstract), is the knowledge of PHENOMENA, and comprises the four fundamental departments—Physics, Chemistry, Biology, Mental Science. The ARTS are classified as Mechanical, Chemical, Physiblogical, and Mental.

The work of Auguste Comte, entitled 'Cours de Philosophie Positive' (1830-42), is both a classification of the sciences as a whole, and a minute sub-division of each, according to certain fundamental principles.

He first draws the primary distinction between the Abstract and the Concrete Sciences, which he fully illustrates. The Abstract Sciences, being the fundamental or departmental branches of Knowledge, are susceptible of an orderly classification on the principles of Generality, Simplicity, and Independence.

Accordingly, he commences with MATHEMATICS, whose truths

are the most general of all, and wholly independent of the truths of any other science, while all other sciences depend upon it. Its sub-divisions are, the more abstract portion called Number, including Arithmetic and Algebra, and the applications of these to Space (Geometry), and to Motion (Rational Mechanics).

His second science is ASTRONOMY, which is the embodiment of the Law of Gravitation. It receives this position because the carrying out of gravity requires Mathematics alone, while the phenomenon of gravity is a prelude to Physics.

Then come, in order, PHYSICS, CHEMISTRY, BIOLOGY, and SOCIOLOGY, whose mutual position and interior arrangements are governed by the same ideas of growing dependence and complexity, and decreasing generality.

In addition to the singling out of Astronomy as a leading science, Comte's arrangement has these two farther peculiarities, namely, the omission of Psychology, as a separate departmental science, (it being appended to Biology, under 'Cerebral Functions,') and the inclusion of Sociology, or the Science of Society, as a fundamental department.

Mr Herbert Spencer, in his recent work entitled 'The Classification of the Sciences,' has criticised the scheme of Comte, and propounded one of his own, which he has developed with circumstantial minuteness. He deals exclusively with the Theoretical sciences.

Mr. Spencer's fundamental idea is the important distinction of Abstract and Concrete, which he expresses in a variety of forms; it is the distinction between the Relations of phenomena and the Phenomena themselves, between the Analytical and Synthetical; it is the separation of one or a few sequences from the total *plexus* of sequences; the wholly, or partially *ideal* as contrasted with the *real*.

Not content, however, with a simple binary division according to this leading contrast, Mr. Spencer proposes a three-fold division, by interpolating between the extremes a middle class partly Abstract and partly Concrete, to be termed Abstract-Concrete. The three classes are ABSTRACT, ABSTRACT-CONCRETE, and CONCRETE. The only way that this is competent is to subdivide the Abstract, according to degrees of Abstractness. 'Concrete' has no degrees; it means the phenomena taken in their full totality, or individuality,—Stars, Mountains, Minerals, Plants, Animals; and there can be but one way of giving these totals, one mode of concreteness. There may, however,

be various degrees of the analytic separation—more or less abstract relations indicated; quantity and form are more abstract than weight, hardness, colour, life.

The **ABSTRACT** Sciences by pre-eminence, are those that deal with the most abstract of all relations—*Space* and *Time*. Without affirming that Space and Time are intrinsically mere forms, conceived by us without any particular things extended and enduring, Mr. Spencer holds that they have *acquired* this character by hereditary transmission, and that we do actually possess them in their empty condition, or apart from any concrete embodiments. Hence, whatever relations subsist with reference to these great conceptions, are the most abstract that the mind can possibly entertain; they are pure and proper abstractions; their hold of the concrete world has been almost, if not altogether, severed. *Space* is the abstract of all relations of co-existence. *Time* is the abstract of all relations of sequence. Now there are two sciences that are occupied with these abstract relations of co-existence and of sequence—*Logic* and *Mathematics*; which accordingly form a class by themselves, being removed from the next class by a wider interval than separates the members of that class from one another.

Proceeding from the blank Forms of existence, to Existences themselves, from the *relations* of phenomena, to the *phenomena*, we find two divisions, having different aspects, aims, and methods. In fact, we have the distinction of Abstract and Concrete carried out, without the same absolute divorce as in the previous class. Mr. Spencer illustrates the distinction thus:—Every phenomenon is a manifestation of force, usually a combination or complication of forces (the course of a projectile depends upon at least three forces). We may study the forces either in separation, or in combination—the *factors* or the *product*. On the one hand, neglecting all the incidents of special cases (say of falling bodies), we may aim at educing the laws of the common force (gravity) when it is uninterfered with. On the other hand, given all the incidents of a phenomenon (as a river), we may seek to interpret the entire phenomenon, as a *product* of the several forces simultaneously in action. The truths reached through the first kind of enquiry, though concrete inasmuch as they have actual existences for their subject-matter, are abstract as referring to the modes of existence *apart from one another*.

Mr. Spencer thinks it proper to point out farther that the *abstract* must not be confounded with the *general*. Each has its peculiar signification; 'abstract' means detachment from

particulars; 'general' means *manifestation* in numerous cases. The law of uniform rectilinear motion is *abstract*; but it is never realized in any particulars, consequently it is not general; while rotation on an axis is very *general*. Accordingly, he disapproves of Comte's expression 'decreasing generality,' as belonging to the phenomena of the successive sciences,—Mathematics, Physics, &c. This criticism indicates a point worth noting, but as regards Comte's remark it might easily be evaded. There can be no abstraction without a prior generalization; the abstract law of rectilinear motion, is a generalization of the very highest order stating what would happen in every case when a body is projected into space and left to itself. The other kind of generality is something more special and concrete, in fact, much less of a generality than this great primary law.

The Sciences, then, that treat of the forces of phenomena, as analyzed and handled in separation, are the ABSTRACT-CONCRETE Sciences; as Mechanics, Physics, Chemistry. The sciences that view phenomena in their aggregate, or their full actuality, are Concrete Sciences; such are Astronomy, Geology, Biology, Psychology, Sociology, &c.

A few words now as to the more precise definitions and divisions of the leading departments, on which hang various points of logical interest.

ABSTRACT SCIENCE considers, first, what is common to all Relations, and next, what is common to each order of Relations. Between each kind of phenomenon and certain other kinds of phenomena, there exist uniform relations. It is a universal abstract truth—that there is an unchanging order among things in Space and in Time. This is the most abstract truth of all, the subject-matter of the highest division of Abstract Science. It has sub-divisions. First, and next in abstractness, are the connexions of things in Space and Time, irrespective of the things connected. This is the subject-matter of *Logic*, where the nature and amounts of terms related are not considered, but only the relations themselves. The other sub-division takes in Quantity or amount, without any farther qualities. This is *Mathematics*, which is a statement of laws of quantity apart from any real things, that is, as occupying Space and Time. This statement is made upon certain *ultimate units* occupying definite positions in Space and in Time. The divisions of Mathematics follow according as the units are simply separate, or according as they are both separate and equal; the one gives birth to an indefinite Calculus (applied

in Statistics), the other to the Definite Calculus, whose subdivisions are *Arithmetic*, *Algebra*, and the *Calculus of Operations*. When the computation of units refers to occupation of Space, the subject is *Geometry*. When Time is introduced, we have *Kinematics* and the *Geometry of Motion*.

1 So much for the sciences of pure Abstraction. The second class, the ABSTRACT-CONCRETE, are occupied with the general laws of Motion, Matter, and Force, in their disentanglement from the concrete phenomena, where they re-act upon, and modify one another. In Mechanics, for example, which is one of the sub-divisions, the laws of motion are expressed without reference to friction and resistance of the medium (?). So in Chemistry, another sub-division, the laws are viewed upon substances absolutely pure, such as Nature rarely supplies.

The partition of this group is conducted on the same principle as in the former group. A distinction is drawn between Force considered *apart from its modes*, and Force considered *under each of its modes*,—a more abstract, and a less abstract department. The first part contains a statement of the Laws of Force, as deducible from the fundamental principle of the Persistence of Force, together with the theorems of the Composition and Resolution of Forces. The second part comprises *Molar Mechanics* or Molar Forces (Statics, Hydrostatics, Dynamics, Hydrodynamics), and *Molecular Mechanics*—including the properties and states of matter (Physical), and Chemistry; together with Heat, Light, Electricity, and Magnetism. [The arrangement is a questionable one, in so far as Chemistry is interposed *between* the Physical properties and states of bodies, and the agencies—named Heat, Light, &c].

The division of Abstract-Concrete Science is thus co-extensive with what we have formerly termed Inorganic Physics.

The third great group, the CONCRETE SCIENCES, as repeatedly stated, embrace the totalities of phenomena. Astronomy is placed in this group. The meaning is, that the astronomer does not stop short after generalizing the laws of planetary movement, such as they would be if there existed only one planet; he solves this abstract concrete problem, *as a step* towards solving the concrete problem of the planetary movements as affecting one another. The 'theory of the Moon' means an interpretation of the Moon's motions, not as determined simply by centripetal and centrifugal forces, but as perpetually modified by gravitation towards the Earth's equatorial protuberance, towards the Sun, and even towards Venus,—forces daily varying in their amounts and combinations. So the

geologist does not confine himself to the separate elements—water-action, fire-action, he aims to interpret *the entire structure of the Earth's crust.* And, in Biology, if different aspects of the phenomena of Life are investigated apart, they are all helping to work out a solution of vital phenomena in their entirety, both as displayed by individual organisms and by organisms at large. The interpretation is no longer synthetical but analytical.

These explanations premised, the enumeration of subjects in the Concrete division is as follows:—First, and most general of all, are the Universal Laws of the continuous Re-distribution of Matter and Motion. Next follows the application of these to actual Matter. As applied to the Celestial Bodies (1) treated as masses, it is *Astronomy*; (2) as made up of molecules—*Astrogeny* (Solar Mineralogy and Solar Meteorology). On the earth, the same actions result in *Mineralogy*, *Meteorology*, *Geology*; when causing organic phenomena, they make up *Biology*, which has various sub-divisions, terminating in *Psychology* and *Sociology*.

Such is the outline of Mr. Spencer's scheme. By way of criticism, the following remarks may be offered.

In the first place, objection may be taken to his language, in discussing the extreme Abstract Sciences, when he speaks of the *empty forms* therein considered. To call Space and Time empty forms, must mean that they can be thought of without any concrete embodiment whatsoever; that one can think of Time, as a pure abstraction, without having in one's mind any concrete succession. Now, this doctrine is in the last degree questionable. For although we might concede the hereditary predisposition to fall into these conceptions, we do not thereby affirm that they can be bodied forth without any concrete examples whatever. We might rather say with Kant, and the later *a priori* schools, that *when particulars are given they start forth into full view*. This much is certain, however, that without a very wide and familiar converse with particulars, the exceedingly abstract relations of these Abstract Sciences, are wholly incomprehensible to any human being. The extreme generalities of Logic, in order to be intelligible, need perpetual reference to particulars. The same is true with the first elements of Mathematics, which are the foundations of all the rest.

Mr. Spencer's account of the subject-matter of Logic, the first of all the sciences, is so extremely general that we can hardly discover what is the precise scope he assigns to it.

From its position, however, it must be viewed as Theoretical Logic purely; under which there would be included the fundamental aspects of all knowledge—Difference (Relativity) and Agreement (Generality), the Laws of Consistency, Mediate Inference, the Uniformity of nature; and the various deductions or consequences of those primary facts. These are points common to all sciences, and may therefore precede them all. At the same time, it should be remarked that the ascertaining of these very high generalities has been a great inductive effort, considerably aided by the special study of the human mind, or the science of Psychology. This observation slightly qualifies Mr. Spencer's statement that none of the truths of the third group are of any use to the problems of the second, while the second group are of no use to the first.

It may be farther noticed that, notwithstanding the strong terms employed to contrast the Abstract with the Abstract-Concrete Sciences, the contiguous subjects of each show but a narrow boundary line. The geometry of Motion, the last of the Abstract Sciences, comes very close upon the Universal Laws of Force, the first subject of the Abstract-Concrete group.

These considerations, if they have any weight, tend to invalidate the alleged distinction between Abstract and Abstract-Concrete Sciences, a distinction without an adequate difference. Practically, however, the matter is of no moment. The succession of subjects would probably be regarded as the same, and the manner of sub-dividing and treating them would be very much the same with or without this particular boundary. Mathematics must precede Mechanics; and Logic, conceived in its high theoretic aspects, may claim to precede Mathematics.

A much more serious dispute arises out of Mr. Spencer's proposed boundary line between the Abstract-Concrete and the Concrete Sciences. No one ever drew the line as he has done it. The Concrete Sciences have always been typified by the so-called Natural History Sciences—Mineralogy, Botany, Zoology, Geology—and by Geography. These are Sciences whose marked features are Classification and Description. They deal with large collections of objects, which they arrange and describe by means of careful generalization.

It is, therefore, with a little surprise that we find inserted among Concrete Sciences, not merely Astronomy, but the whole of Biology, in which is included Psychology. Certain parts of these subjects would be properly concrete; as Celestial Geography (under Astronomy); and the Races and Characters of men (under Psychology.)

Let us consider how the case stands with Astronomy. This science, since Newton's time, is avowedly based on Theoretical Mechanics. Newton, in the First Book of the Principia, which may be pronounced Abstract Mechanics of the purest type, went far beyond Mr. Spencer's limits to an Abstract-Concrete Science. These limits, indeed, are not a little arbitrary. We can suppose a science to confine itself *solely* to the 'factors,' or the separated elements, and never, on any occasion, to combine two into a composite third. This position is intelligible, and possibly defensible. For example, in Astronomy, the Law of Persistence of Motion in a straight line might be discussed in pure ideal separation; and so, the Law of Gravity might be discussed in equally pure *séparation*—both under the Abstract-Concrete department of Mechanics. It might then be reserved to a *concrete* department to unite these in the explanation of a projectile or of a planet. Such, however, is not Mr. Spencer's boundary line. He allows Theoretical Mechanics to make this particular combination, and to arrive at the laws of planetary movement, *in the case of a single planet*. What he does not allow is, to proceed to the case of two planets, mutually disturbing one another, or a planet and a satellite, commonly called the 'problem of the Three Bodies.' This problem is not to be touched in Theoretical Mechanics, but to be remanded to the Concrete Science of Astronomy. Yet, if we are allowed to combine the *two* factors—projectile motion and gravity to one centre—why may we not take in an additional factor, a second gravitating body? The difference is not between single factors and their combination, but between *two grades of combination*.

In point of fact, such a line is never drawn. Newton, in the First Book of the Principia, took up the problem of the Three Bodies, as applied to the Moon, and worked it to exhaustion. So writers on Theoretical Mechanics continue to include the Three Bodies, Precession, and the Tides. Nor is any reason apparent for making the break that Mr. Spencer suggests. Increasing complicity of deduction and calculation attends the inclusion of new factors, but this special difficulty is not supposed to take the subject out of an abstract department and to insert it in some concrete department.

Again, Mr. Spencer remarks that in works on Mechanics, the laws of motion are expressed without reference to friction and resistance of the medium. Turning to 'Thomson and Tait's Mechanics,' we find the Laws of Friction introduced, with a reservation of the purely Experimental results to the

department called Properties of Matter. In Newton's Second Book, and in all works of similar compass, the operation of a Resisting Medium is handled.

The law of the radiation of light (the inverse square of the distance) is said by Mr. Spencer to be Abstract-Concrete, while the disturbing changes in the medium are not to be mentioned except in a Concrete Science of Optics. We need not remark that such a separate handling is unknown to science.

Mr. Spencer's illustrations from Chemistry are especially at variance with usage, while it is difficult to reconcile them with reason. Chemistry is an Abstract-Concrete Science. What does this mean? The reply is, the chemist is never satisfied with the crude substances of nature, but first purifies them, and ascertains the properties in the pure state. This, of course, is a necessary precaution. But if the insinuation be, that Chemistry does not give, or ought not to give, the properties of any impure substance, or any alloy or mixture, the fact is quite different. Every chemical writer describes all the prevailing species of carbon, including pure and impure kinds; the same with iron, and with every substance found in important varieties. Why should it be otherwise? There is no dereliction of logical principles in stating the properties of the iron ores, in connexion with iron. The same thing may be repeated in Mineralogy, but is not out of place in Chemistry. Again, no writer on Chemistry ever omits to describe the Atmosphere, which is the actual or concrete combination of Oxygen, Nitrogen, &c.

It may be noticed in addition that a substance *purified* is obviously not a substance in the *abstract*. Virgin gold, and the purest diamond are still objects in the concrete.

These remarks on Chemistry pave the way for the consideration of the place assigned to Biology among the *Concrete* Sciences. Now, Biology is a science of increasing complication; living bodies are subjected to all the Physical and Chemical Laws, and to Biological Laws in addition: so that a rose is a more complicated object than a diamond. But the objects of Chemistry and the objects of Biology are equally concrete, so far as they go; the simple bodies of chemistry, and their several compounds, are viewed by the Chemist as concrete wholes, and are described by him, not with reference to one factor, but to all their factors. The isolation of the one property, named Chemical combination, which would be an abstract handling of bodies in the chemical point of view,

must be considered to be impracticable; at all events it is never done. We may doubt whether anything would be gained by attempting it. But, whatever abstractive operation of this kind is possible in Chemistry, might be repeated in Biology; there might be general laws—isolated factors—of life, as well as of inorganic matter. If so, to place one of these two leading departments among Abstract Concrete Sciences, and the other among the proper Concrete departments is to make a distinction without a sufficient difference.

Nor is it possible to justify the placing of Psychology wholly among Concrete Sciences. It is a highly analytic science, as Mr. Spencer thoroughly knows. The totality of mind is separated into factors, each discussed in isolation, before they are brought together. There are many strictly abstract discussions to show the difference between the effect of a motive (as selfishness) acting in ideal purity or separation, and the same motive, combined with many others, in the concrete human being. But the force of the remark would appear to be dissipated if all the laws of Psychology are to be considered as expressions of the concrete facts of mind.

A separation may be temporarily made between the purely theoretical and deductive treatment of a science, and the experimental treatment. In Theoretical Mechanics, (as Hydrodynamics), the laws of a resisting medium may be inferred and computed from primary assumptions as to the nature of fluid particles; while, on the other hand, the subject may be investigated by experiments, as in gunnery. But the science is not completely presented unless both are taken account of together: the theoretical deductions have to be confronted, checked and verified, by the experimental results, in order to have any standing as laws of the department.

Yet another method is possible. A subject, as, for example, Astronomy, may be exhaustively handled in a separate treatise; wherein there shall be brought together from every department whatever bears upon the celestial bodies. This would be a highly *mixed* department, yet not, on that account, a strictly concrete science. It would be full of the most abstract discussions, witness the 'Mécanique Céleste' of Laplace. It would draw contributions from various sciences, besides its parent science, Mechanics; it would introduce Optics, Heat, Magnetism, and Chemistry; yet it would not treat the heavenly bodies as Minerals are treated in Mineralogy, or Plants in Botany. It would have many practical bearings; in fact, it would have considerable claims to be a Practical Science. Any

scientific department exhaustively treated would eschew purity, and draw contributions from many sources.

Thus, it appears that Mr. Spencer, in abandoning the usual partition of the sciences, into the departmental or fundamental sciences, on the one hand, and the concrete or derived on the other, has abandoned the more real distinction in search of a fanciful and untenable boundary line of the Abstract and the Concrete. We see reason still to abide by the old specification of the Concrete Sciences, typified by Mineralogy, Botany, Zoology, Geology, &c. These sciences have marks peculiar to themselves; they are the *classificatory* and the *descriptive* sciences. They embrace large collections of individual things, which have to be classified, and to be described as concrete wholes. Moreover, they contain no new fundamental operation of nature; every variety of natural agent has been previously exhausted in the departmental sciences—Mathematics, Physics, Chemistry, Biology, Psychology.

B.—THE PROVINCE OF LOGIC.

It is contended by some logicians that the Province of Logic is Formal Reasoning and Thinking; by which they mean mainly the Syllogism, and what is subsidiary thereto. They would exclude everything that refers to the Matter, that is to say—Induction, and the greater part of Definition and Classification.

We have, however, just grounds to complain that the distinction of FORM and MATTER is too vague and unsteady to constitute a clear line of demarcation between the two departments of Evidence—Deductive and Inductive. It will be expedient for us, therefore, to ascertain what precise meanings, if any, can be assigned to these phrases.

Perhaps the most thorough and consecutive account of the severance of Formal Logic from Material Logic is that contained in the Introduction to Mansel's edition of Aldrich. In that work, the author adduces every consideration that is of any avail in widening the distinction in question.

Adverting to the first question raised in the definition of Logic, namely, whether it be a Science or an Art—whether it is principally theoretical or principally practical—Mr. Mansel holds that, in its essence, it is speculative or theoretical, and, in its accidents, practical. There would be a body of principles or laws, although no one cared to apply them to the discipline of the mind, or to the improvement of the thinking faculties.

Nevertheless, the science is susceptible of application to practice; it may be brought to bear on our intellectual processes. Such is its scope as expressed in the second part of Whately's definition—the *Art of Reasoning*; which definition, however, as regards the word 'Reasoning,' Mr. Mansel, in common with Hamilton and Mill, objects to as narrowing the province too much. Even as a Formal Science, Logic includes the processes named Apprehension and Judgment, and these not as mere aids to Reasoning, but as independent acts of thought. Accordingly, Mansel agrees with Hamilton in substituting for 'Reasoning,' with suitable qualifications, the larger term 'Thought.'

He then proceeds to lay out the distinction between the Form and the Matter of the thought. His first indication of the difference is to this effect:—Thought may violate *its own laws*, and so destroy itself; something may be set up that turns out wholly *unthinkable*. On the other hand, a Thought may be perfectly consistent with itself, but at variance with facts of *experience*; which, although quite thinkable, would be empirically illegitimate, or *unreal*. [This is the distinction between Self-Consistency—Immediate or Equivalent statements, and Inductive or matter-of-fact certainty].

The next remark is that there must be *material data* in order to thought of any kind, even formal thought; there must be concrete experience of things external and things internal, in order to understand even a syllogism. But the materials being given, there is a vital difference between two modes of using them. The distinction of *Presentative* and *Representative* thought is an aid here; the distinction between the individual concrete things—a building, a man, a star, and the generalities or concepts—height, figure, brightness, which we may form by the comparison of the concrete objects. The consideration of the Matter is the reference to the individual things; the consideration of the Form is the general concept, or representative thought. [So far we have the ordinary distinction between Concrete and Abstract, only it is apparently pushed to a kind of Conceptualism; there being implied that the concept, or notion, is something more than an *agreement* among individuals. If it be true that a notion is unthinkable, except as one or more individuals, the 'Form' is still 'Matter,' only in a somewhat different arrangement].

But farther, the *thinking process* may be distinguished as material or formal. It is *formal* when the matter given is sufficient for the product derived, with no other addition but

the act of thinking. It is *material* when the data are insufficient, and the mind has to take in more matter, in the act of thinking. Given the attributes, A, B, C, we can think them as co-existing in an object, without any fresh appeal to facts; which is *formal conceiving*. [This is quite intelligible too; all the operations of Arithmetic are formal in this sense; we pronounce six times four to be twenty four, without an appeal to pebbles or coins, or any real objects. We have put together from primary realities a machinery that can operate independently of the realities].

As conditions of formal conceiving, are laid down the laws of Contradiction and Identity. We must not introduce Contradictory attributes—A and not-A. The author is a little more obscure as regards the condition of Identity. Thought, he says, is representative of all possible objects; but Intuition (cognition of the individual, as opposed to Thought, or the general) must be conscious of differences; every object of intuition is marked off, limited, and individualized; it is *itself* and no other. To this circumstance corresponds the Law of Identity, 'A is A'; 'every object of thought is conceived as itself.' A somewhat novel rendering of that well-known Law of Thought.

These laws are the key to logical *conceiving* (Conception is the first logical product). Next, as to formal *judging*, or the forming of Judgments. Affirmation takes place when one concept is contained in another; Negation, when one contradicts another. Here, too, are involved the laws of Identity and Contradiction.

Finally, as to *reasoning*. This is formal when the given judgments are connected by a middle term, under such conditions of quantity and quality that the *mere act of thought* necessarily elicits the conclusion. If there be required any addition to the data, the consequence is material. Formal Mediate reasoning, no less than Immediate inference, is achieved through the laws of Identity (for affirmative syllogisms), and of Contradiction (for negative syllogisms). In the immediate inferences of Opposition [Obversion] and Conversion, there is a further demand for the subordinate law of Excluded Middle.

Thus, then, if a thought professes to be based on formal grounds, to be guaranteed by the laws of thought alone, its pretensions can be adjudicated on by Logic; if it professes to rest on sensible experience, or on suppressed premises, it must come before another tribunal.

It is, of course, open, the author remarks, for any innovator

to propose an extension of boundaries, by the inclusion of the Matter of propositions; but he does so in the teeth of Kant's demonstration, that *'a criterion of material truth is not only impossible, but self-contradictory'*. Moreover, the attempt to enlarge the field renders impossible the assigning of any definite field whatever.

We are interested to know in what way Mr. Mansel makes good these very strong allegations. The steps are these.

(1) The Aristotelian or Formal Logic seeks the laws whereby the *mind thinks*; the Baconian seeks the laws whereby the phenomena of *outward things take place*; that is to say the one refers to mind, the *ego*, the other to matter, the object, or *non-ego*. Consequently, the one enquiry is the interrogation of self-consciousness, the other is an examination of external nature.

Such is Mr. Mansel's first position. It seems to involve some confusion of ideas. We strongly doubt whether the contrast of Formal Logic and Inductive Logic can be reduced under the contrast of Subject and Object, or Mind and Matter.

For one thing, the study of Mind, or Psychology, is, in modern times, universally considered to be properly Inductive. How can we reach the important laws of Mind—such as Relativity, Association of Ideas, the operation of the Feelings, and the Will—except by observation and induction of the facts of self-consciousness, occasionally aided by external indications.

Again, the laws of Thought, called Identity, Contradiction, and Excluded Middle, apply alike to the outer world and to the mind. If so, they may be gathered from either source. Probably, however, the supposition is that these laws are got at without investigation; that they work themselves out without being expressly studied. We unconsciously and irresistibly declare that the same thing is not at the same instant white and black; just as we walk without thinking how we walk.

These invincible tendencies of the mind, if such there be, are no doubt facts of our mental nature: but so is our belief that Nature is uniform, or that every effect must have a cause; on which reposes all Inductive investigation. In both cases, the mind is the instrument, although the material may be sometimes mental phenomena and sometimes phenomena of the outer world. Deduction and Induction have equally their seat in laws of the thinking mind; and have equally, for their field of operation, both mind and matter.

(2) The next position is this—The Aristotelian laws are laws of thought *as it ought to be*; the Baconian laws are laws of

nature *as it is*. The author adds, as explanatory and synonymous statements,* what seems to involve a new and distinct idea, namely, that the one rest on their own evidence, the other on the evidence of the facts concerned.

To this we may reply that 'thought as it ought to be' is certainly not confined to Formal Reasoning. Wherever we think wrong, and have to be put right, we are in the domain of 'thought as it ought to be.' Lord Bacon's inductive logic professed to substitute right thinking for wrong. We commit fallacies of Deduction and of Induction equally; and if Logic does not put us right upon both, it must be for some other reason than the one now assigned.

The addendum given, professedly to explain the above position, namely—that the Aristotelian laws are self-evident, and irreversible in thought, while the Baconian laws are inductions from facts and contingent or reversible—is merely a re-statement of the general thesis as between self-evident or necessary truth, and inductive or contingent truth.

(3) The third position is that the Aristotelian Logic proceeds from the *law* to the *facts*, constructing types or generalities, and rejecting what does not conform thereto; while in the Baconian Logic, the procedure is from the *facts* to the *law*, rejecting every law that does not account for the facts. This is a direct opposition of *Method*.

Now, we may readily grant this position. But what is its bearing on the question in dispute? The methods are different, but both are methods of arriving at truth; both may be alike in want of precautions, and if so, both may, so far as appears, equally receive attention from the logician.

(4) The fourth position is perhaps the most remarkable. It is this: *Law*, in the Aristotelian system, implies a *consciousness of obligation*; whereas, in the Baconian system, *Law* means only *uniform sequence*.

Here is that confusion of thought, so well pointed out by John Austin, in connexion with the term 'Law,' whereby there is introduced into the order of natural phenomena the notion of authority and obedience. Law, as regards the order of nature, whether in mind or matter, is purely figurative; it is applicable merely as expressing *uniformity of sequence*; the Ethical and Political definition—a rule set by intelligent superiors to intelligent inferiors, accompanied by the infliction of pain on neglect—cannot be transferred to the sequences of nature, whether mental or material; the application to these contains only the single incident of law—uniformity. There

can be no moral right or wrong in Logic, except only in so far as we are all morally bound to seek the truth, an obligation extending equally to truth Deductive and to truth Inductive.

(5) A fifth position maintained by the author is, that, in the field of Thought, the *cause* is the conscious self; the *effects*, the thoughts produced by that self, through its own power, and under its own laws. To which we may reply, that both causes and effects are equally self, equally mental, but not thereby radically contrasted, in manner of investigation, with external nature. Cause and effect in mind must be discovered inductively, if at all. Should the sequences be very prominent, little attention may suffice for their discovery; but that does not alter the method of proceeding.

So much is Mr. Mansel carried away by the application of the term Law, in its Ethical sense, to the process of thinking, that he censures Mr. Mill for applying 'physical causation' (meaning uniformity of sequence, ascertained by induction) to the moral and intellectual world; as if there ever was any other mode of discovering the facts and laws of mind than the same processes, observation, and generalization, that apply to the material world. In short, he brings us round by a series of verbal ambiguities to the question of Free-Will and Necessity, which becomes thus a principal turning-point of the controversy as to whether Logic should, or should not, be confined to Deduction.

The combined force of these five positions does not appear to establish either of the two allegations, namely (1) that a criterion of material truth is not only impossible, but self-contradictory, or (2) that to enlarge the field of Logic, is to assign it no definite field. We shall not here attempt a direct reply to the first, inasmuch as the exact basis of inductive truth will be fully considered in another place. (APPENDIX D.) The second allegation is a challenge to assign a definite boundary to Logic, while over-stepping the limits of the Formal Logic.

Mr. Mansel puts so much more stress on the Theoretical than on the Practical side of Logic, that he would not be satisfied with a reply based on the practical side. Let us enquire, then, whether a Theoretical Logic, embracing Induction, could be laid out and so circumscribed as not to be confused with any other scientific department, such, for example, as Mathematics, Physics, or Psychology.

In the INTRODUCTION, we have indicated a field of Theoretical Logic, according to the larger meaning of the Province; and

in APPENDIX A, we have given Mr. Spencer's survey of the field in the same larger meaning. In summary, we may repeat the topics.

I. THE LAWS OF CONSISTENCY, or Equivalence of Propositions, commonly understood as the Laws of Thought. These give necessary (in the sense of analytic) inferences. They also give, in the view of Hamilton and Mansel, the basis of the Syllogism.

II. THE LAWS OF DEDUCTIVE or Mediate Inference, as represented by the *Dictum de omni et nullo*. This we hold to be more than mere Self-consistency, or Equivalence. It might be called *Mediate Consistency*, the consistency of a conclusion with two conjoint premises, as contrasted with the consistency of an equivalent transmutation of a single proposition. Mr. Mansel would hold that this consistency is necessitated and self-evident; and such an impression is not uncommon with thinkers generally. In opposition to that view, we have contended that nothing less than the induction of material instances would justify the conclusion.

III. THE LAW OF THE UNIFORMITY of Nature, which is the basis of all material truth, and of all induction; consequently the basis of the syllogistic axiom of mediate consistency. The consideration of this law may well precede the ordinary sciences, for it is an assumption running through them all. It may, therefore, receive its first announcement in the science that deals with the criteria of all truth, namely, the separate science of Logic. It is followed out into a series of formulæ, known as the Inductive Canons, which, in their own sphere, may be compared with the syllogistic forms, in the Deductive sphere.

Now, it seems to us, that a science may be constructed so as to include the Laws and Formulæ of Immediate Consistency, Mediate Consistency, and General Uniformity, without transgressing the sphere of any other science. It need not run into Mathematics, the kindred Formal Science; it need not trespass on the Physical Sciences, merely because it considers the postulate necessary to them all, that is, Uniformity; it need not run into Psychology, although it derives from that science the explanation of the ultimate nature of Knowledge, as Difference and Agreement. And there does not appear to be any other conterminous region.

But we cannot concede to Mr. Mansel that Logic is essentially, or in the main, a theoretical science, and only incidentally practical. We contend that the science would never have been called into existence, but for its supposed practical utility.

Indeed, the same might be said of its splendid giant brother, Mathematics. However agreeable and recreative to some minds may be the contemplation of this great creation of ages, yet, but for the necessities and difficulties of measurement, it would never have been heard of. Mr. Mansel supposes a race of intelligent beings, subject to the same laws of thought as we are now, but incapable of transgressing these laws; and declares that in the presence of such a race, the Logic of the Formal Concept, Judgment, and Syllogism, would remain the same. Unfortunately even for the illustration, there is a fallacy of Relativity in the very statement of the case. To a being that never committed an error, truth and error would be alike unmeaning; to appreciate the valid moods of the syllogism, as contrasted with the invalid, such a being would have first to be told of an erring race, capable of confounding the two. Only after Adam fell did he know good and evil; only by committing fallacies is any one competent to understand Logic.

Postponing for a little the enquiry into the practical utility of the Inductive extensions of Logic, we shall advert more particularly to the distinction of Form and Matter, on which so much stress is laid in the present dispute. To some Formal Logicians the distinction does not appear in all respects satisfactory. Thus, Dr. Thomson (*Outline of the Laws of Thought*, § 15) remarks:—‘The philosophic value of the terms matter and form is greatly reduced by the confusion which seems invariably to follow their extensive use. Whilst one writer explains form as ‘the mode of knowing’ an object, another puts it for ‘distinctive part,’ which has to do with the being or nature of the thing rather than with our knowledge of it; where it means ‘shape’ in one place, which is often a mere accident, in another it means ‘essence;’ so that it may be brought to stand for nearly opposite things. I will add, that probably there is no idea which these terms represent that cannot be conveniently expressed by others, less open to confusion.’

Mr. De Morgan says:—‘When it shall be clearly pointed out, by definite precept and sufficiently copious example, what the logicians really mean by the distinction of form and matter, I may be able to deal with the question more definitely than I can do at this time.’ (*Cambridge Transactions*, vol. X Part II. p. 8.) Again, ‘The truth is, the mathematician as yet, is the only consistent handler of the distinction, about which,

nevertheless, he thinks very little. The distinction of form and matter is more in the theory of the logician than in his practice; more in the practice of the mathematician than in his theory.' (Syllabus, p. 48).

Hamilton illustrates Formal Truth in Mathematics thus:—
 "To the notions of Space and Time, the existence or non-existence of matter is indifferent. If matter had no existence, nay, if space and time existed only in our minds, mathematics would be still true; but their truth would be of a purely *formal* or *ideal* character,—would furnish us with no knowledge of objective realities.' (Logic II, p. 66). But, in another place, he quotes, with approbation, from Esser, a passage to the effect that truth consists not in any absolute harmony of thought, but in the correspondence of our thoughts with their objects. 'The distinction of formal and material truth is thus not only unsound in itself, but opposed to the notion of truth universally held, and embodied in all languages.' (Logic I. 106). And again (Reid's works, p. 687), he remarks of Reid's criticism on the Predicables, that Reid, like our British philosophers in general, was unaware of the difference between the *Logical* or *Formal*, and the *Metaphysical* or *Real*. The Predicables are *forms* or *modes* of predication, and not *things* predicated: in the language of the schools, *second notions*, not first.'

Let us adopt Mr. de Morgan's suggestion, and refer to Mathematics for examples of Form, in the opposition to Matter. In so doing, however, we are merely taking up an old subject under a new name. In Mathematics, we have the most complete development of reasoning by Symbols, called also Abstract reasoning. There will be other opportunities for examining the special processes of Mathematics (LOGIC OF THE SCIENCES, *Mathematics*). For the present, let us note what bears upon the question before us. The abstractions of Mathematics, like all other abstractions, are embodied in concrete instances; the Form is always given in some kind of Matter. But the matter needed is so very spare and attenuated, that, by a stretch of language, we may say it is no matter at all. Yet, the circles of Euclid are circles of printer's ink; they have colour and a definite size. If we compare them with the round shield of Achilles, or a gorgeous centre ornament in the roof of a palace, we may describe them as void of matter and substance; but they have their own substance, nevertheless.

The symbols of Arithmetic (still more, of Algebra) are material, although their peculiar shape has nothing representa-

tive in it. They are the signs of *concrete facts*—one, two, three—which are inconceivable by us, except in concrete instances. The simplest material will answer the purpose—bread crumbs, pebbles, mud specks; but we must have, in the mind, a series of discrete impressions, derived somehow or other; even thoughts would do; but we find it easier to work upon things of sense. Without some concrete basis, we cannot possess in thought any number whatever. This is merely to repeat the received nominalistic view of Abstract Ideas.

There is, however, an important step that can be made in Mathematical Reasonings, whereby we can altogether leave out of sight the concrete things (which is to refrain from realizing the very meanings of the numbers that we are handling). We can devise *rules of operating* upon the symbols, which, when duly constructed and checked by the proper precautions, will give us the same results as actual experiments upon the concrete numbers. Having constructed our decimal notation, we can base upon it a multiplication table, containing equivalent formations of numbers; and by mere force of memory, recalling these symbolical equivalents, we can perform operations of multiplying, without thinking of the concrete numbers at all. In getting out the product of 94 by 116, we can leave the world of numbered realities out of view for the time: coming back to it only when the product has to be practically turned to use.

Now, by this dwelling among symbols, and rules and signs of operation, we are as far away from Matter, or things in the concrete, as we can possibly be. If anything represents pure Form, the multiplication table does. The higher operations of Algebra keep us for longer periods withdrawn from concrete reality; but the principle is the same. The symbolical creations are more numerous, the rules of operation more complicated; the operations themselves more protracted; yet there is nothing new in the principle of working.

The question then arises, Do these rules of operation upon symbols bear out the pretensions of Formal Logic, as to the self-evident, necessary, and non-material character of Formal Thinking? Are all such rules, in their origin, completely withdrawn from the tests of concrete experience, as they are in the working? The full answer to this question is the theory of Deductive Reasoning in general, and of Mathematical Reasoning in particular. It is enough here to make two observations. First. If it be true, as the *a posteriori* thinkers maintain, that the final axioms of all Mathematics,—on which repose the

rules for Arithmetical sums, for Algebraic equations, and for Geometrical demonstrations,—are inductions from experience, then these various rules of operation have, after all, a purely 'material' source, and are not evolved by the mind in abstract or formal thinking.

• But secondly. It is notorious and undeniable, that the rules of operation, before they are trusted to, are tried and checked by the results. A great many of them are so paradoxical, so unpromising, and even repugnant, to the ordinary mind, that they are admitted only because of their being instrumental in bringing out true results (as proved by reference to the matter). Who would put faith in such a rule as 'minus multiplied by minus gives plus,' unless fully assured by concrete trials that it leads to correct conclusions? The impossible quantities of common Algebra, the infinitesimals of the higher Calculus, have been a perpetual stumbling-block, as regards their Form; their sole justification is the test of actual facts.

Seeing how many ingenious tricks can be played upon us by formulas and formalities, the most unexceptionable in their appearance, there probably is not a single rule in the whole compass of Mathematics that any reflecting person would trust to merely as a 'Law of Thought,' without an appeal to the matter by actual trials. The reason why we are so confident in these rules, is that their verification is so easy, and has been so complete. But in the absence of verification, we should be very chary indeed in admitting such rules as the multiplication and division of fractions, vulgar and decimal, the extraction of the cube root, and the like. We have often been deceived by more plausible formalities than these; *dolus latet in generalibus*, is true of all alleged 'Laws of Thought.'

The same remark as to the necessity of inductive verification applies to Logical Forms. Not one of the valid moods would be received by mankind upon formal evidence alone. The *dictum* seems very evident, the *nota notæ* even more evident; but the *nota notæ* conducts us most plausibly to false conclusions, until by examination of the actual cases we have laboriously fenced it with circumlocutions and qualifications.

When we examine carefully the various processes in Logic, we find them to be material to the very core. Take *Conversion*. How do we know that, if No X is Y, No Y is X? By examining cases in detail, and finding the equivalence to be true. Obvious as the inference seems on the mere formal ground, we do not content ourselves with the formal aspect. If we did, we should be as likely to say, All X is Y gives All Y is X; we

are prevented from this leap merely by the examination of cases.

Again, the laws of Hypothetical Equivalence are dependent on our knowledge of the material circumstance called Plurality of Causes, but for which the formal directions as to Hypothetical Inference would be quite different.

Mr. Mansel complains that the rules of Definition commonly given in logical treatises are extra-logical; that is, they step out of Form into Matter. The charge is well founded; the writers obviously felt that Definition, confined within the narrow limits of the Formal, would be a very meagre affair. What would be logical defining in strict form? Why, this. A Formal Definition consists in giving, as the marks of the thing defined, the marks of some higher Genus, together with the Difference. We have, then, these forms:—The Genus together with the Difference (in Connnotation) is the Species; the Species *minus* the Difference is the Genus; the Species *minus* the Genus is the Difference. This is the whole theory of Defining, according to Formal Logic; and it is worth nothing.

Still more would a logic of Classification, to be of any value, trench upon material considerations. Logical Division is another name for classification. The rules of Logical Division are Formal, but they have to be held in check by the matter, otherwise they may lead us astray.

It may be maintained that Deduction and Induction are properly *continuous* operations; they are the parts of one whole. Within certain small limits, Deductive processes are possible, upon rules of symbolical operation solely, these having been well fenced by a study of the matter; but real deduction, the extension of a principle to new cases, supposes an examination of the cases in their concreteness or actuality, exactly as in the inductive generalization of the rule. The judge who applies the law must look to the matter; he must not commit paralogisms of form; but he cannot stop short at mere formal correctness.

Within the Inductive sphere, we might construct rules of Formal operation, such as ought to commend themselves to a rigid formalist. Thus, A, B, and C, being joint causes of an effect X; if A be reduced in amount, B or C must be correspondingly raised to keep up the effect; if A be increased, the others are so far dispensed with, and so on. These are easy mathematical considerations, which we know to be correct

generally, and can therefore use formally without regard to the matter. •

• But the question at issue cannot be adequately stated, unless we view Logic as a Practical Science. If its practical character is conceded, the propriety of extending the Province rests upon the utility of rules for Induction. The presumptions in favour of such rules are these :—

• First. It is admitted that Aristotle included in his scheme both Deduction and Induction, however imperfect may have been his view of their respective spheres, and however inadequate may have been his handling of Induction. Thus, the testimony of the Founder of Deductive Logic is opposed to its exclusive pretensions.

Secondly. In the table of Fallacies, sketched by Aristotle, and retained by the scholastic logicians, with slight modifications, there are comprised Fallacies of the Matter, and of these some are fallacies of Induction (*non causa pro causa, &c.*). From this we may infer, that, in the opinion of logicians generally, people are liable to commit mistakes in regard to matter, no less than in regard to form. We may infer further, that it is not useless to give a reminder of these material and inductive mistakes, which is, in fact, a Logic of Induction.

Thirdly. The scholastic period was marked by an almost exclusive attention to the formal or Syllogistic part of Logic. At the revival of letters and philosophy in the 15th and 16th centuries, public opinion revolted against the narrowness of the conception, and found a spokesman in Bacon, who inaugurated, amid very general applause, a Logic of Induction. For the last two centuries and a half it has been the pride of both physical and metaphysical philosophers to call themselves his disciples as regards the methods of pursuing science and philosophy.

Fourthly. The renovated Physics, or Natural Philosophy, of Galileo and Newton was accompanied with a professed Logic of Induction—the famous *Regulæ Philosophandi* prefixed to the Third Book of the *Principia*. These rules, meagre as they are, were a guiding star in physical research to the enquiries of the 18th century.

Fifthly. In the present day, when physical science has been so far advanced as to exemplify sound methods of procedure, the most distinguished physical philosophers still feel and acknowledge the need of a systematic guide to research, for the more abstruse and subtle departments. The Introduction to

Natural Philosophy, by Sir John Herschel, and the History and Logic of the Inductive Sciences, by the late Dr. Whewell, are testimonies to this want.

Sixthly. Since the publication of the work of Mr. John Stuart Mill, in which the Inductive Logic is methodized with a completeness previously unknown, applications have been extensively made of the Inductive canons to the Experimental Sciences. The investigations of Medical science have especially profited by Mr. Mill's teaching; a higher and surer standard of evidence has taken the place of the loose methods of reasoning formerly prevalent.

Seventhly. The Science of Politics is an equally striking example. The valuable work of Sir George Cornwall Lewis on the 'Methods of Observation and Reasoning in Politics,' makes perpetual reference to the Inductive Logic of Bacon, Herschel, Whewell, and Mill, and only once or twice alludes to Formal Logic, although the author's education was such as to incline him to view that department with the utmost possible favour. He complains strongly of the wide-spread abuse of the Method of Agreement (the *enumeratio simplex* of Bacon) in Politics, as in other subjects; and endeavours by precept, and by example, to counterwork the vicious tendency.

Eighthly. Sir William Hamilton occupies a considerable portion of his Course on Logic (*nine* Lectures out of Thirty-six), with Modified Logic, in which he considers Truth and Error, on the material side; Observation; Induction; the Credibility of Testimony; and various other points related to the acquisition and communication of knowledge. The plan of his course would have allowed him, without contradicting his views of the Province of Logic, to have gone as minutely as Mr. Mill does, into Induction, and the operations subsidiary to Induction, such as Classification and Naming.

Dr. Thomson, in his Laws of Thought, follows the example of Hamilton, in the enlargement of the Province. In Part IV., entitled 'Applied Logic,' he considers (shortly) the Search for Causes, the Inductive Methods, Definition, Analogy, Chance, Classification, Fallacies generally, and the Division of the Sciences.

C.—ENUMERATION OF THINGS.

The Classification of Names (p. 61) leads by a natural transition to the Classification of Things. Moreover, in order to establish the most generalized propositions, we must possess correspondingly generalized Notions.

The totality of Existing Things may be divided in various ways, under different principles of classification and division. We may partition the whole universe into Celestial Bodies and Terrestrial Bodies; into Minerals, Plants, Animals; into Solid, Liquid, Gas; into Ponderable and Imponderable; into the Four Elements of the ancients, which division crudely gives the three states of matter, and the imponderables—Heat, Light, &c. Lastly, we may make a division into Matter and Mind. These various modes of sub-dividing the totality of things are useful for their special purposes. The purpose of the Logician is to arrive at a division that will correspond to the distinct methods of enquiry, so as to partition the field of knowledge according to the best division of intellectual labour.

We begin by re-stating, as an essential preliminary, the principle of Universal Relativity, by which all objects of knowledge are two-sided, or go in couples. This statement is necessary to obviate the error, committed by Aristotle and others, of placing 'Relation' in an inferior or subordinate place in the classification. If Relation is recognized at all, it is fundamental and independent; everything comes under it, it comes under nothing. The supreme position given by Logicians to the 'Law of Contradiction' is a mode of admitting this primary fact.

1. The deepest of all Relations is OBJECT and SUBJECT, commonly called Mind and Matter, the External World and the Internal World.

When we pass from being engrossed with pleasure or pain to the consciousness of some extended thing, as a tree, we are affected with a marked shock of difference; we have made a transition the broadest and deepest that the mind can ever pass through. These typify the two ultimate or final modes of the human consciousness; they mutually constitute each other, on the principle of Difference or Relativity; they cannot, therefore, be resolved one into the other, or into any more fundamental experience. The contrast must be accepted as the chief division of all things, on the principle of dividing upon the maximum of difference. One portion of knowledge we term the *Object* world, the Extended World, and, less correctly, Matter, and the External World. The other portion we call the *Subject* world, the Unextended Mind, and, less properly, the Internal World. Indeed, when we talk of these two departments as dividing between them the universe of existence, we are using fictitious and unmeaning language; the ultimate universe, according to the law of Relativity, is a

couple; the highest *real* grouping of things is this *two-fold* grouping, called Object and Subject, &c. These are the proper *summa genera*. Existence is a mere name.

II. OBJECT has been variously represented and analyzed. Some have contended that it is an ultimate fact, given in our earliest consciousness. Others have resolved it into simpler states of the mind. The different views on this subject belong to the Metaphysical and Psychological question called the 'Theory of External Perception.' We here assume that the notions expressed by 'Object' and 'Subject,' can be analyzed, and we give one mode of the analysis. Object means (1) *what calls our muscular and bodily energies into play*, as opposed to passive feelings; (2) *the uniform connexion of definite feelings with definite energies*, as opposed to feelings unconnected with energies; and (3) *what affects all minds alike*, as opposed to what varies in different minds.

(1) The greatest antithesis existing among the phenomena of our mental constitution is the antithesis between the Active and the Passive; the muscles (with the out-carrying nerves) being the bodily instrument for the one, the senses (with the in-bringing nerves) being the bodily instrument for the other. To this fundamental antithesis we are able to link the opposition of Object and Subject. Although developed by other circumstances, the contrast appears to be rooted in our greatest Psychological contrast.

(2) The circumstance of our feelings being definitely changed with definite active exertions on our part is a most notable accompaniment of our objectivity. When we move across a room, and feel our optical prospect definitely changing with every step, and always going through the same definite changes with the same movements, we put this experience in contrast with feelings that fluctuate when we are perfectly still, and have no relation to our movements; as the stages of an illness, the periodic sensations of hunger and fatigue, and the various passions and emotions.

(3) It is a characteristic of the Object world, that different persons are affected in the same way. Those definite changes of sense, accompanying definite movements, as in walking down a street, or in entering a room, arise in each person alike; the other class of feelings—hunger, fatigue, fear—run a different course in different persons.

These are probably the main features of the fundamental contrast of Subject and Object; other subsidiary circumstances have been pointed out, but their discussion is not suitable to this place.

III. The **SUBJECT** is explained by what has been said of the **Object**; it concerns our passive states; our feelings not definitely changed with definite energies; and the states wherein different persons vary in the same circumstances.

IV. There are attributes common to **Object** and to **Subject**, and attributes special to each.

Notwithstanding the fundamental contrast of these two experiences, we can affirm some attributes of both. Thus, within the sphere of each, we are variously affected; we recognize object distinctions and subject distinctions. So we identify and compare object facts with one another, and subject facts with one another. From the very nature of human knowledge, these possibilities of discerning agreement and difference must hold in both departments. Hence:—

First. The contrasting attributes of **LIKENESS** and **UNLIKENESS** belong equally to **Object** states and to **Subject** states. We identify and discriminate magnitudes, forms, colours, &c., which are object facts; we identify and discriminate pleasures, pains, volitions, ideas, which are subject facts. Hence, affirmations of likeness or of unlikeness may apply to every kind of knowledge whatsoever. Being in fact the fundamental circumstances that define and constitute knowledge, such affirmations are analytical propositions.

Secondly. **QUANTITY** or **Degree** belongs to both states. This is **Agreement** and **Difference** in one important fact or feature, called more and less; the states of the subject mind are all of varying amount or intensity, as well as the states of the object consciousness, which we call object properties—size, weight, hardness, &c. We may and do predicate quantity, therefore, of everything knowable. The laws of **Quantity**, of which **Mathematics** is the complete development, pervade all modes of existence. It is true that numerical calculations are mostly confined to object properties—as space, dimensions, weight, and so on; we have no numerical ratios in pleasures and pains. This circumstance, however, which is a great drawback to the science of mind, is not due to the absence of degree from mental phenomena, but springs from our inability to set up an exact common standard of degree in the states of the mind, and to take precise measures according to that standard. We are conscious of inequalities in our pleasures, emotions, and desires, but we have a difficulty in fixing the degrees in an understood expression, such as may be communicated to others, and permanently recorded.

It is usual to specify the leading modes of **Quantity** under

Intensity, Duration, and Extension : the last being a mode special to the object. Intensity and Duration⁴ apply in both regions of phenomena. *Intensity* is usually marked with regard to each special property—intensity in colour, heat, pressure, &c. *Duration*, which is a degree of continuance, is more commonly abstracted from things, and enters into that great and all-comprehending generality, called TIME, to be noticed more fully under next head.

Thirdly. The great and important contrast named Co-existence and Succession is found in both departments of phenomena.

Co-existence is not an ultimate experience of the mind. We begin with modes of *Succession*, which are developed into Co-existences.

To the mind, which, with very slight qualification, can attend to but one thing at a time, all distinctive states of consciousness are successive. Succession is the law of our mental being. The succession may be rapid or slow, which supposes the estimate of duration above noticed. In succession is grounded the important fact called Number or Discrete Quantity, as opposed to the measure of continuance, or Continuous Quantity. We identify groups of successions as twos, or threes, fours, and so on. Thus the forms and modes of Quantity are involved in the modes of succession of our sensations, feelings, and thoughts.

Duration and Succession (with Number) thus belong alike to states of the Object and states of the Subject. The element of TIME, which is duration and succession generalized to the utmost, and reduced to a common measure, is a property of both worlds; a circumstance that has been noticed from the very beginning of philosophy.

The predicate of Succession also involves order of priority, which can apply to object and to subject states equally.

Co-existence is an artificial product, a peculiar mode of succession, which in its highest form is Simultaneity in Space, or Extension, a property of the Object sphere exclusively. There attaches to Mind an inferior mode of Co-existence, the co-existence of two or more awakened sensibilities at one moment of time.

Of Attributes common to both spheres, we have thus Like-Unlike, Quantity, Succession, Co-existence; but as the predication of Like-Unlike in the widest sense is, from the nature of knowledge, a purely identical proposition, we need state only Quantity, Succession, and Co-existence. These are the

three attributes assumed as distributing knowledge into different heads of Logical Method.

V. The attributes special to the OBJECT, are as follows :—

(1) *Extension*.—This property is the fundamental circumstance of the object world, the one fact common to whatever is not mind, or not subject. When we are in a purely subject state, as a pleasure or a pain, we have no consciousness of extension or space. The distinction between extended matter and the unextended mind, explicitly made in the 5th century, A.D., was the commencement of correct views of mind and matter.

Psychologically considered, Extension is a mode of our active or moving energies, assisted by our senses. Motion is essential to the consciousness of things as extended. Extension is a real property whether with or without matter ; as scope for motion, even empty space is an actuality. The total of the Extended World is sub-divided into Extended Matter and Extended Space without matter.

(2) *Resistance, Inertia, Momentum, or Force*.—This is the characteristic property of Extended Matter, in its opposition to an Extended void. The putting forth of our energies in the peculiar mode called Resistance is perhaps the simplest situation that we can be in, as regards the active side of our being ; hence, resistance may be considered our fundamental consciousness of the object world. Resistance is Matter ; the giving way of resistance, followed by movement, is Space. In no subject state have we the peculiar sensibility called force, energy, or resistance ; where that feeling is present, we apply the name matter.

Extension and Inertia are the two generic facts entering into the long known group of attributes called the *primary qualities of matter* ; the radical and identifying peculiarities of the so-called external and material world. Still, these are in close association with other properties, based on passive sensibility, or sense proper, as colour, tactile feeling, &c. (*secondary qualities*) ; which properties, of themselves, would not be object properties, but become so by their dependence upon the object class.

(3) *Colour*.—The pure and proper sensibility of the eye, the susceptibility to mere light, is not properly an object fact. The conjunction of the feeling with visual extension (the muscular sensibility of the eye), and with locomotion, is necessary to give objectivity to light and colour. Our notion of the extended or simultaneous in space is based on movements, but

filled up and defined by our optical sensibility to varieties of light. Our feelings of illumination are definitely connected with definite movements and in that way comply with one of the grand conditions of objectivity.

(4) *Touch*.—The commonly recognized sense of Touch is a compound of muscular energy with pure skin sensibility. This last, or touch proper, is scarcely ever separated from the fundamental experience of Force or Resistance (we may make the separation by supporting the outstretched arm or leg). Hence, touch is adopted and embodied among object properties. The tactile effects, called hard, soft, rough, smooth, are qualities of Matter.

Sight and Touch are the senses most completely incorporated with our activity, or with our object experience. The remaining senses have a looser connexion with our energies, but, so far as connected, we rank their indications among object qualities.

(5) *Sound*.—Mere noise might be a form of simple subjectivity. When related to movements, as when steadily increasing or diminishing with our locomotion, it falls into a connexion with objectivity. So regularly is this connexion observed, that the fact is enrolled among properties of matter.

(6) *Odour*.—An exact parallel to Sound. The objectivity of odour is established by its definite changes under definite movements on our part.

(7) *Taste*.—There is here a compound of a peculiar sensibility—the proper gustatory feeling—with touch proper; whence it comes readily into the object sphere.

(8) *Heat and Cold*.—This property needs no other comment than the foregoing remarks on Sound and on Odour.

The various organic sensibilities of our body—Digestion, Respiration, &c.—have a strongly subject character; yet, they contract object relationships whenever they are definitely changed with definite movements, as when we connect repletion with taking food, or suffocation with impeded breathing. But, in so far as they suggest no activities, or attitudes of energy, they are pure subject states, modes of self-consciousness.

These are the various sensible properties of the *species* 'matter' in the *genus* 'extended'; they are the modes of primitive sensibility that we call material. There are other properties of a more subtle and abstruse kind, arrived at by the help of our intellectual processes—such as we call Attractions, Repulsions, Molecular structure and arrangements—which are necessary to completeness in the enumeration.

The Sciences of the so-called External world are occupied with the various attributes now described. One portion of Mathematics is occupied with quantity in Extension; Mechanics embraces the essential fact of Matter, together with its other incidents; Physics and Chemistry include Light, Sound, Odour, Heat, &c.

VI. The attributes special to the SUBJECT are the defining marks or essential attributes of Mind—*Feeling*, *Will*, and *Thought*. All these are in full antithesis to the great object facts, as above detailed.

Of *Feelings*, the greater part are pleasures and pains, which are our most unequivocal types of subjectivity. We never confound two such things as comfortable warmth, and lifting a chair; the heterogeneous is at its utmost stretch in such a contrast as this.

Our states of *Will*, or Volitions, have a purely subject origin, namely, our feelings, with outcomings in the object sphere. The two departments are here, as often happens, in close proximity, but are not therefore confused. Voluntary action is always reckoned a special characteristic of mind. For, although it is activity, directed often upon material things, yet its origin in the pleasurable and painful modes of sensibility gives it an indelible stamp of the subject.

Our *Thoughts*, Ideas, or Intellectual states, have in them a considerable amount of object reference; still there is a broad distinction between Sensations and Ideas, in the circumstance that the one class is, and the other is not, connected with definite bodily movements. The succession of our sensations is in uniform accordance with our locomotive and other movements; the succession of our thoughts is totally different. Hence, although our ideas are the reflexion or repetition of our sensations, yet their manner of occurrence assimilates them with subject states.

In the complex fact called Sensation, we have incessant shiftings of the scene, from the object to the subject. A sensation, as cognisant of extension, resistance, colour, &c., is an object fact; as a pleasure or a pain, it is subject. Now, unmistakeable as the contrast is, wide as is the chasm, we may leap it a great many times in a minute; we flutter to and fro, between the pleasurable consciousness of a sensation, and the intellectual measure of it as a thing of size, form, or colour.

The sciences of the Subject World have thus to deal with our Feelings, Volitions, and Thoughts. They have, moreover,

to draw the delicate boundary line between the two worlds, to divide the spheres, where they become entangled.

If it were now asked what, in the final analysis, is the nature of predication, we are able to affirm—*Attributes* of the *Object*, and *Attributes* of the *Subject*, declared as related in *Quantity*, as *Co-existing* or as *Successive*.

VII. SUBSTANCE is not the antithesis of all Attributes, but the antithesis between the fundamental, essential, or defining attributes, and such as are variable or inconstant.

From the relative character of the word Attribute, the fancy grew up that there must be a *substratum*, or something different from attributes, for all attributes to inhere in. Now as anything that can impress the human mind—Extension, Resistance, &c., may be, and is, termed an attribute, we seem driven entirely out of reality, if we would find a something that could not be called an attribute, and might stand as a substance.

But 'substance' cannot be rendered by non-entity. The antithesis that we are in search of is made up without so violent a supposition. Substance is not the absence of all attributes, but the most fundamental, persisting, inerasible, or essential attribute or attributes in each case. The substance of gold is its high density, colour, lustre, &c.—everything that we consider necessary to its being gold. Withdraw these, and gold itself would no longer exist: substance and everything else would disappear.

The substance of *Body* or *Matter*, is the permanent, or essential fact of Matter—Inertia or Resistance. This is the feature common to everything we call Body—whether Solid, Liquid, or Gas; the most generalized, and therefore the defining property of Matter. The remaining attributes of matter vary in each separate kind; they *make* the kinds or specific varieties—air, water, rock, iron, &c. The real distinction is thus between the *Essence* and the *Concomitants*, the *Invariable* and the *Variable*, the *Genus* and the *Species*.

The substance of *Mind* is no other than the aggregate of the three constituent powers—Feeling, Will, Thought. These present, mind is present; these removed, mind is gone. If the three facts named do not exhaust the mind, there must be some fourth fact; which should be produced and established as a distinct mode of our subjectivity. The substance would then be four-fold. But the supposition of an 'ego' or 'self,' for the powers to inhere in, is a pure fiction, coined from non-entity,

by the illusion of supposing that because attribute applies to something, there must be something that cannot be described as an attribute.

Mr. Mill, as the result of his analysis, gives the following as an enumeration and classification of all Nameable Things:—

- '1st. Feelings, or States of Consciousness.
- '2nd. The Minds which experience those feelings.
- '3rd. The Bodies, or external objects, which excite certain of those feelings, together with the powers or properties, whereby they excite them; these last being included rather in compliance with common opinion, and because their existence is taken for granted in the common language from which I cannot prudently deviate, than because the recognition of such powers or properties as real existences appears to be warranted by a sound philosophy.

'4th, and last. The Successions and Co-existences, the Likenesses and Unlikenesses, between feelings or states of consciousness. Those relations, when considered as subsisting between other things, exist in reality only between the states of consciousness which those things, if bodies, excite, if minds, either excite or experience.

'This, until a better can be suggested, may serve as a substitute for the abortive Classification of Existences, termed the Categories of Aristotle. The practical application of it will appear when we commence the inquiry into the Import of Propositions; in other words, when we inquire what it is which the mind actually believes, when it gives what is called its assent to a proposition.

'These four classes comprising, if the classification be correct, all Nameable Things, these or some of them must of course compose the signification of all names; and of these, or some of them, is made up whatever we call a fact.' (Logic Book I., Chap. III).

The Categories of Aristotle.

We owe the Categories to the opposition made by Aristotle to Plato's Realism of Universals. Plato viewed *Eins* or Real Being as belonging only to Universals separated from their particulars; they only being permanent as contrasted with the Generated and Perishable. Aristotle held, on the contrary, that Real Being attached only to the Particulars; that certain varieties of Being might be predicated of an individual—*Hoc aliquid*, That man, This horse, &c.—but that no Being had

any reality apart from the individual. The varieties of Being that might thus be predicated of a particular 'individual, he enumerated in a schème known as the Categories (*κατηγορίαι*, *Predicamenta*). They are as follows :—

1. *ὄντις*—*Substantia*—Substance.
2. *Ποσόν*—*Quantum*—Quantity.
3. *Ποιόν*—*Quale*—Quality.
4. *Πρός τι*—*Ad aliquid*—Relation.
5. *Ποῦ*—*Ubi*—Location.
6. *Πότε*—*Quando*—Period of Time.
7. *Κεῖσθαι*—*Jacere*—Attitude, Posture.
8. *Ἐχεῖν*—*Habere*—Equipment, Appurtenance, Property.
9. *Ποιεῖν*—*Facere*—Active Occupation.
10. *Πάσχειν*—*Pati*—Passive Occupation.

Mr. Mill points out the more obvious defects of the Categories considered as an enumeration of Things.

'The imperfections of this classification are too obvious to require, and its merits are not sufficient to reward a minute examination. It is a mere catalogue of the distinctions rudely marked out by the language of familiar life, with little or no attempt to penetrate, by philosophical analysis, to the *rationale* even of those common distinctions. Such an analysis, however superficially conducted, would have shown the enumeration to be both redundant and defective. Some objects are omitted, and others repeated several times under different heads. It is like a division of animals into men, quadrupeds, horses, asses, and ponies.'

Hamilton endeavours to obviate this last objection, by casting it into a scheme of successive grades of subordination. His elucidation is as follows :—'Being (*τὸ ὄν*, *ens*) is primarily divided into *Being by itself*, (*ens per se*), and *Being by accident*, (*ens per accidens*). Being by itself corresponds to the first Category of Aristotle, equivalent to Substance: *Being by accident* comprehends the other nine,' but is, I think, more properly divided in the following manner :—*Being by accident* is viewed either as absolute or as relative. As absolute, it flows either from the matter, or from the form of things: if from the matter,—it is *Quantity*, Aristotle's second category. If from the form, it is *Quality*, Aristotle's third category. As relative, it corresponds to Aristotle's fourth category *Relation* : and to Relation all the other six may be reduced.

The arrangement would stand thus :—

| | | |
|------------------|---|------------------|
| I. Substance (1) | | |
| II. Attribute | { | Quantity (2) |
| | { | Quality (3) |
| | { | Relation (4) |
| | | Place (5) |
| | | Time (6) |
| | | Posture (7) |
| | | Appurtenance (8) |
| | | Activity (9) |
| | | Passivity (10) |

There is no evidence that Aristotle saw the division in this light; if he had done so, he might have adverted to the misplacement of 'Relation,' which, if it includes any of the others, equally includes them all; Substance and Attribute, Quantity, Quality—are all relationships. Still, the arrangement is useful as showing how some of the worst defects may be remedied, and as an aid to remembering the list. The four first are easily remembered; the remaining six (under Relation) may be cast into three couples—Place and Time, Activity and Passivity, Posture and Possession or Appurtenance.

The Categories do not seem to have been intended as a classification of nameable things, in the sense of "an enumeration of all kinds of Things which are capable of being made predicates, or of having anything predicated of them." They seem to have been rather intended as a generalization of *predicates*, an analysis of the final import of predication, including Verbal as well as Real predication. Viewed in this light, they are not open to the objections offered by Mr. Mill. The proper question to ask is not—In what Category are we to place sensations, or any other feelings or states of mind, but—Under what categories can we predicate regarding states of mind? Take, for example, Hope. When we say that it is a state of mind, we predicate 'substance:' we may also describe how great it is ('Quantity'), what is the quality of it, pleasurable or painful ('Quality'), what it has reference to ('Relation'). Aristotle seems to have framed the Categories on the plan—Here is an individual: what is the final analysis of all that we can predicate about him?

The proper comparison of the Categories is to the Predicables, and to the Import of Propositions, or the Universal Predicates. Comparing the Categories with the Predicables, we see that through both runs the distinction between Fundamental and Concomitant, Essential and Accidental. The four predicables, *genus*, *species*, *differentia*, *proprium*, are predications of 'substance:' *accidens*,—*concomitance* (συμβεβηκός) embraces

all the categories except substance. Other categories than substance might be *propria*, or predication deduced from the essence of the subject; but it is probable that Aristotle, in speaking of 'fundamental' and 'concomitant' in connection with the categories, meant to include *propria* in the category of substance. Probably Aristotle's list of *propria* had been smaller than the list that could be made out now. Secondly, if we compare the Categories with the Universal Predicates (*Co-existence, Succession, Quantity*), we see that the Categories are more superficial and less ultimate than the later analysis. The category of 'substance' (if we do not include *propria*) belongs to the department of Verbal predication: the remaining Categories are Real predicates, corresponding to the final analysis of propositions. As such an analysis, they are open to the objection of not being ultimate; for example, the predication concerning 'space' and 'time' may regard 'co-existence' or they may regard 'succession.' More than this, they are not adapted to any logical purpose; they cannot be made the basis of logical departments.

While these comparisons show the bearings of the categories as regards Logic, it should be kept in mind that their original purpose was simply to exhaust the possible predicates regarding an individual, and not either to exhibit a classification of nameable things, or to analyze the import of propositions with a view to the arrangement of logical departments.

D.—THE UNIVERSAL POSTULATE.

The theory of Demonstration supposes that we come at last to something that cannot be demonstrated. Demonstration is the referring of a fact to a higher generality, already established; to demonstrate such higher generality would be to find some principle still more general; a few steps must lead us to something that is absolutely final, something whose evidence is not demonstrative, something believed in without extraneous support.

The edifice of demonstration is not complete until we clear out these ultimate foundations, and state distinctly the nature of the certainty attaching to them. Let us then ask what are the facts to be received without proof, as underivable, undeducible, undemonstrable?

In probing to the deepest foundations of knowledge and certainty, there has often been a confusion of two classes of primary facts—the Logical and the Psychological. By the Logical *primordia* are meant the indemonstrable assumptions

at the foundation of all demonstrable truth; by the Psychological, are meant the elementary sensibilities of the mind, whence our complex intellectual products are evolved by growth, aggregation, or association. What the logical foundations are, will be stated fully in this note; the Psychological foundations are the primary sensibilities arrived at in an ultimate analysis of the mind—such as Resistance, Motion, Colour, Sound, &c. There may be a partial coincidence of the two classes of ultimate data; but the coincidence is not necessarily total; and each must stand on its own grounds. The propriety of an Analysis of the mind needs to be established by evidence; hence it must appeal to some first principles different from itself; so that the priority belongs to the Logical foundations of our knowledge.

The phrase 'Universal Postulate,' proposed by Mr. Herbert Spencer, to express the ultimate foundations of certainty, is adopted from Euclid. While the subject-matter is quite different in the two applications, there is this common feature, that in both something has to be begged on one side and granted on the other; one person cannot force another person into the admission. The basis of all reasoning is something mutually conceded between the different reasoners. When an opponent accepts a certain first principle, and declares that he will abide by all its consequences, we may compel him to accept whatever we can show to be a consequence; but we have not the same fulcrum with the first principle itself.

In reviewing the modes of stating the primary assumptions, we may commence with the so-called *Laws of Thought*—Identity, Contradiction, and Excluded Middle. These, however, are too limited for our purpose. As explained in this work, they are laws of Consistency and Equivalence; the Formal Logicians suppose them to include also Syllogism, or Mediate Consistency; by no one are they held as furnishing a criterion of material truth.

Hamilton has put forward 'the testimony of Consciousness' as the ultimate and infallible criterion of certainty. He expresses the reference to consciousness in these three maxims or precautions:—

(1) That we admit nothing, not either an original datum of consciousness, or the legitimate consequence of such a datum.

(2) That we embrace all the original data of consciousness, and all their legitimate consequences; and—

‘(3) That we exhibit each of these in its individual integrity, neither distorted nor mutilated, and in its ‘relative place, whether of pre-eminence or subordination.’ (Reid’s Works, p. 747).

Stated in general terms, this criterion seems unimpeachable. But when we come to specific enquiries, we are aware of its vagueness and uncertainty. Our present consciousness must be admitted to be our present consciousness; when we feel hungry, we have the fullest certainty that we are hungry. The question, however, arises—what does consciousness say to facts in the past, and to facts in the future. And strange as the thing may appear, people may differ as to what things we are actually conscious of, as will be seen presently.

Mr. Spencer expresses the Universal Postulate under the form of the Inconceivability of the Opposite. The only reason assignable, he says, for our primary beliefs, is the fact of ‘invariable existence tested by an abortive effort to cause non-existence.’ When the opposite of an assertion is utterly unthinkable by us, we can do nothing but receive that assertion as true.

The difficulties attending the employment of this test are these :

First. The examples that are most in its favour are cases where the opposite is a self-contradiction. I cannot think that I do not at present exist, because the two suppositions are incompatible; the attempt is a violation of the law of consistency. So,—‘Motion cannot be thought of without an object that moves being at the same time thought of’ is an instance where the two statements give the very same fact; ‘motion’ and ‘a thing moving,’ are two slightly different phrases for an identical conception. The opposite is pure self-contradiction.

Now, for all such instances, a postulate of self-consistency would answer the same end as a postulate of unthinkableness of the negation.

Secondly. In assertions where there is not mutual implication but difference in things conjoined, the inconceivableness of the disjunction has arisen from unremitted experience, or indissoluble association. This is the case with extension and colour; we cannot think of an object as extended without thinking it as of some colour; the visible form, although a different fact from colour, has always been embodied in an optical impression of colour. Again, ice cannot, without great difficulty, be thought of but as cold; the visible appearance

of ice and the sensation of warmth are repugnant because of the strong opposing association.

The same remark applies to the (proper) Axioms of Mathematics. The iteration of them in experience creates an almost indissoluble link of thought in their favour. We are practically unable to think their opposites. So with the Logical Axiom of Mediate Consistency.

Now, with regard to this class of beliefs, it is an open question, whether the stress should be laid upon the acquired inconceivableness of the negations, or upon the circumstance that has brought about the inconceivableness, namely, the unbroken iteration of the facts. Whether are we to lay hold of the primary condition, or of its consequence or concomitant? There seems to be a presumption in favour of the primary condition, namely, the unbroken experience.

Mr. Spencer himself attributes our inability to conceive the opposites of axioms and other strong beliefs to the experience of the race accumulated and transmitted to us. 'Objective facts are ever impressing themselves upon us; our experience is a register of these objective facts; and the inconceivableness of a thing implies that it is wholly at variance with the register.'

Thirdly. There are propositions admitted by us to be universally true, but whose opposites we can well conceive. Such is the law of gravity. We can easily suppose that law to be suspended. The reason in this case is, that although the greater number of unsupported bodies fall to the ground, some do not; smoke and dust may be seen ascending. We learn to regard these as exceptions, but they prevent us from having an overpowering strength of association between the absence of solid support and the descent of a body to the ground.

Fourthly. Some examples given as unquestionable applications of the principle of Inconceivableness are denied by a whole school of thinkers. Both Sir W. Hamilton and Mr. Spencer maintain that we are under the necessity of believing the Persistence of Force; that we cannot conceive either Matter or Force as absolutely created or absolutely destroyed. It is under the first kind of inconceivableness (where the opposite is a self-contradiction) that this case is brought; there is no attempt to affirm it on unbroken experience. The self-contradiction, however, is by no means apparent; Force is one thing, and its commencement or termination is seemingly a different thing. That aspect of Force whereby, in communicating itself, it loses the numerical equivalent of what is

communicated, becomes familiar to us after we are educated in mechanical facts; and we are then prepared to receive the doctrine of Persistence. But prior to this experience, which, to be sure, is requisite to a clear and precise cognition of Force, we can form a conception of force beginning we know not how, and ending we know not how. We are not at first struck with any self-contradiction in force arising out of no prior force; the contradiction that we discover at last is a contradiction of our experience.

A still more doubtful example is furnished by the question of questions—Material Perception, which Mr. Spencer upholds in its popularly received form, on the authority of the test of inconceivableness of the negative. Mysterious as is the consciousness of something out of consciousness, we are, he says, obliged to think it. 'The current belief in objects as external independent entities, has a higher guarantee than any other belief whatever.' Yet this is the belief that would have remained undisturbed to this hour, but for its glaring *self-contradiction*, first exposed by Berkeley, and since by others. (See, in particular, Ferrier's Review of Berkeley). Any test of belief that guarantees this assumption must needs be repudiated by the numerous believers in its self-contradictory character. There is an evident incongruity in laying down, as a universal postulate, what begs the very point in dispute, in a leading controversy.

Fifthly. Mr. Spencer's view, that inconceivableness (where there is no self-contradiction) represents 'the net result of our experience up to the present time,' supposes a theory of the *sources of belief* which is liable to great objections. He considers that our habitual contact with actual things has engrained in our minds an intensity of connexion between the ideas of those things proportioned to the frequency of their recurrence. For example, Space relations are the most iterated of any, and, consequently, our minds are moulded to these with the highest possible tenacity. Next are Matter and Force relations. In this way, as already remarked, our repugnance to form even an idea of the opposites is a proof of the persistence of the corresponding facts. So that, experience and inconceivability of the opposite are convertible statements.

Now, it may be granted that the contact with actual things is one of the sources of belief; but it is not the only nor the greatest source. Indeed, so considerable are the other sources as to reduce this seemingly preponderating consideration to comparative insignificance. The competing elements are

briefly the following :—(1) The innate impetuosity of believing that what is will continue; and (2) The influence of our strong emotions and predilections. Both influences will be illustrated afterwards as prevailing causes of error or Fallacy (Book VI). There should also be taken into account the circumstance that our strength of association does not represent the comparative recurrence of the fact, unless our position is such as to encounter the facts in proportion to their exact frequency. What is most familiar to nature, may not be the most familiar to us. We may not see the world from a central or commanding point of view. The best example of this is our excessive familiarity with one type of causation—the human will; in consequence of which, we represent that as the proper and natural type; whereas, it is an exceptional and narrow instance of causal agency.

There still remains the effect of society in propagating and iterating certain propositions in language; by which iteration, no less than by confronting the facts in our own person, we are moulded to belief in certain doctrines. On the whole, therefore, when the various agencies operating to form our convictions are taken together, the one circumstance assigned by Mr. Spencer is so overborne as to render our strength of belief no just criterion of the facts believed.

Sixthly. Nothing is gained by putting under one head, and subjecting to a common test, two classes of beliefs so distinct, as Self-Consistency and Consistency with Facts. Hitherto, in philosophy, these two departments, under various names, have been kept distinct. The one is known as Formal Truth, Necessary Truth, the Laws of Thought: the other is Material Truth, Contingent Truth, Inductive Certainty. Although the most strongly iterated of the laws inductively arrived at tend to indissoluble associations, and to a difficulty of thinking their opposites—in that way approximating to the truths of consistency, this is a mere incident belonging unequally to things that are alike true. When the inconceivability occurs, a reason can be given for it; and the reason not being always the same, there is no propriety in disguising the deeper differences by the superficial agreement. We are not obliged to have only one Universal Postulate. Should there occur two very different kinds of certainty, neither reposing on the other, our proper course is to assign different postulates.

On these various grounds, we demur to the test of the 'Inconceivableness of the Opposite' as the basis of all certainty, or as the matter that cannot be proved, but must be

asked and granted, before demonstration can begin. We should propose, instead of that test, at least two Postulates, according to the distinction last noted; perhaps more may be requisite.

First and foremost, we should place the Postulate of **CONSISTENCY**, or **Self-Consistency**—the absence of self-contradiction. This is the basis of Immediate Inferences, or Equivalent Forms. It must be conceded as a prime condition of all reasoning, discussion, and intelligent communication. Enough has been said in regard to it.

Secondly, there must be some assumption or assumptions at the foundation of all inferences or conclusions from Experience—some grounds of Material or Inductive certainty. There is much more difficulty in deciding what the postulate should be for the department of *real* inference, or whether a single postulate is enough. We here enter upon a totally new sphere.

In order to guarantee the conclusions of our experience, or to support us in such allegations as—‘water quenches thirst,’ ‘unsupported bodies fall’—there is clearly demanded, in the first instance, a trust in *present consciousness*. We must assume that what we feel, we do feel; that our sensations and feelings occur as they are felt. Whether or not we call this an irresistible belief, an assertion whose opposite is inconceivable or unthinkable, we assume it and proceed upon it, in all that we do. The calling the negation unthinkable does not constitute any reason for assuming it; we can give no reason better than that we do assume it.

The importance of stating this primary assumption is not apparent, till we proceed beyond it. We are carried a very little way into knowledge by the admission taken by itself; we must make some steps in advance, and assume things seemingly precarious in their character when compared with the decisive certainty of immediate consciousness.

It is requisite, in the second place, that we should believe in *past consciousness*, or *memory*. Unless we trust our recollection, our knowledge is limited to what is now present; and we cannot compare two successive experiences, or declare two facts to succeed one another. We have, one moment, the consciousness of thirst; the next moment, we have the consciousness of a certain act called drinking; the next following moment, we have the farther consciousness of relief from thirst. The succession of the three steps is a fact or experience; but we cannot believe it, unless we believe in the

recent fact, given in memory, as well as the present, given in consciousness.

The belief in memory must therefore be postulated. It may be asked, however, are we to believe our memory without limits, or, if not, what are the limits to our belief? If there be any circumstance qualifying or defining the belief, that circumstance should be produced as something more fundamental, and therefore proper to take the place of the assumption that it limits and qualifies. In short, memory must be believed in; yet the postulate of the belief is not wholly independent and isolated, but leans to some extent on another and a different postulate.

Granting, however, that the belief in memory, as well as the belief in present consciousness, is a primary assumption, we next remark that it comes short of our needs. The most authentic recollection gives only what *has been*; something that has ceased, and can concern us no longer. A far more perilous leap remains; the *leap to the future*. All our interest is concentrated on what has yet to be; the present and the past are of value only as a clue to the events that are to come. Now, it is far easier to satisfy us of what has been, than of what is still to be.

The postulate that we are in quest of must carry us across the gulph, from the experienced known, either present or remembered, to the unexperienced and unknown—must perform the leap of real inference. ‘Water has quenched our thirst in the past;’ by what assumption do we affirm that the same will happen in the future? Experience does not teach us this; experience is only what has actually been; and, after never so many repetitions of a thing, there still remains the peril of venturing upon the untrodden land of future possibility.

The fact, generally expressed as Nature’s Uniformity, is the guarantee, the ultimate major premise, of all Induction. ‘What has been, will be,’ justifies the inference that water will assuage thirst in after times. We can give no reason, or evidence, for this uniformity; and, therefore, the course seems to be to adopt this as the finishing postulate. And, undoubtedly, there is no other issue possible. We have a choice of modes of expressing the assumption, but whatever be the expression, the substance is what is conveyed by the fact of Uniformity.

As nature is not uniform in everything, we have to apply a test to discriminate the uniformities from the varieties. There is a uniformity in the manner of animal generation, but

not an absolute sameness in the individuals born even of the same pair. Now experience will not establish uniformity, but it will establish *exceptions to uniformity*; it will sift the natural sequences and enable us to reject all that are not uniform. It does not prove that anything will always be in the future what it has been in the past, but it will prove that some things have been uniform in the past, and others not uniform. It has at least a destructive certainty.

Let us word the postulate thus:—*What has uniformly been in the past will be in the future.* Otherwise, 'what has never been contradicted in any known instance (there being ample means and opportunities of search) will always be true.' In the course of our experience, we have seen a great many promising uniformities break down. Again, we have found instances that have never failed; on such cases, we venture, and it is a mere venture, to predict the future continuance of the same state of things. We go forward in blind faith, until we receive a check; our confidence grows with experience; yet experience has only a negative force, it shows us what has never been contradicted; and on that we run the risk of going forward in the same course.

This assumption is an ample justification of the inductive operation, as a process of real inference. Without it, we can do nothing; with it, we can do anything. Our only error is in proposing to give any reason or justification of it, to treat it other wise than as *begged* at the very outset. If there be a reason, it is not theoretical, but practical. Without the assumption, we could not take the smallest steps in practical matters; we could not pursue any object or end in life. Unless the future is to reproduce the past, it is an enigma, a labyrinth. Our natural prompting is to *assume* such identity, to believe it first, and prove it afterwards.

This third Postulate is, properly speaking, *the Postulate of Experience*. Not only does it involve a hazard peculiar to itself, making a broad line between it and the postulates of present consciousness and of memory, but it seems to remove all the doubts and ambiguities connected with these apparently more facile assumptions. Nothing can be better evidence than present reality, provided we do not mistake an actual consciousness for an inference, or a recollection. This difficulty is got over by comparison of instances, and by the application of general principles, which repose ultimately upon the Great Postulate.

So with Memory. We trust implicitly a recent recollec-

tion ; but as the interval of time enlarges, our trust diminishes. A limit has thus to be prescribed, through a comparison of experiences, followed by an inference from the past to the future, which brings us round again to the assumption of the future from the past. Hence, whichever way we turn, we find this to be the one resting place for the sole of our foot.

• E.²—ARISTOTELIAN AND SCHOLASTIC FALLACIES. •

• The Aristotelian is the basis of all subsequent classifications. It proceeds upon the distinction between fallacies in Language, and fallacies in Thought.

I Fallacies arising in Language (*In Dictione, οἱ παρὰ τὴν λέξιν*). 1. *Aequivocatio*, *Homonymia*, *ὁμωνυμία*; ambiguity in a single term. This is a very comprehensive class of fallacy. One of the examples given by Aristotle illustrates an ambiguity in the word 'necessary.' 'Evil is good, for what is necessary (*τὰ δεόντα*) is good, and evil is necessary.' What is necessary as a means to a desired end is good; but what necessarily results from antecedent conditions may be evil. Whately gives, in his *Logic*, an enumeration of words often used ambiguously in discussion. This task belongs as much at least to the lexicographer as to the logician. Thus: 'Expect' is either what is possible, as that the sun will rise to-morrow, or what is right, as 'England expects every man to do his duty.' 'Old' means either length of duration, or distance of time. As age gives experience, and experience often teaches wisdom, there is a disposition to regard the ancients as wiser than ourselves. To this Bacon replied, '*we* are the ancients;' we inherit the wisdom of the old, and can add to it more experience.

A chief cause of ambiguity is that the signification of words is constantly shifting. The word 'publish' formerly meant 'communicate' or 'show,'—'The unwearied sun publishes to every land.' This is now the legal meaning of publish: to publish a libel is not necessarily to print it, any communication of written libellous matter to another is sufficient. The law still speaks of 'uttering' coin.

'Some' is of interest to the logician, in its two chief senses 'some at least,' and 'some at most,' or some = not none, and some = not all.

The remedy for ambiguity is Definition.

2. *Amphiboly*, *amphibolia*, *ἀμφιβολία*. A sentence may have two grammatical renderings, but by preference suggest the one intended to mislead. This was an occasional trick of the

ancient oracles. 'Aio te, Æacida, Romanos vincere posse,' reads as well whether the Romans are victors or vanquished. 'I hope that you the enemy may slay.'

3. *Fallacia compositionis et divisionis*. Whately defines this fallacy as the use of a term collectively in one premise, and distributively in another. If the term is collective in the major premise, and distributive in the minor, it is a fallacy of division; if the collective is in the minor, and the distributive in the major, it is a fallacy of composition.

| | | |
|--------------------------------|---|-------------------------|
| Five is one number, | } | Fallacy of Division. |
| Three and two are five, | | |
| Three and two are one number. | | |
| Three and two are two numbers, | } | Fallacy of Composition. |
| Three and two are five, | | |
| Five is two numbers. | | |

Aristotle gives a similar division,—*σύνθεσις*, or the possibility of wrong disjunction, and *διαίρεσις* or the possibility of wrong conjunction. His example of *διαίρεσις* is:—

Five is two and three;
Two and three are even and odd;
Five is even and odd.

This would be a fallacy of composition, according to Whately; and Mr. Poste observes that it is not easy to understand exactly Aristotle's distinction, and not worth the trouble.

4. *Fallacia Prosodiae or Accentus*, *προσώδια*. This is of very trifling consequence, and chiefly noticeable because of the different meanings that may be given to a sentence by varying the emphasis. Mr. De Morgan remarks that the commandment, 'Thou shalt not bear false witness against thy neighbour,' is often read with the emphasis so placed as "to suggest that subornation is not forbidden, or that anything false except evidence is permitted, or that it may be given for him, or that it is only against neighbours that false witness may not be borne.' Most of the old examples are mere puns. 'Tu es qui es; quies est requies; ergo, tu es requies.'

5. *Fallacia figuræ dictionis*, *σχήμα λέξεως*. According to Aristotle's view, this fallacy is a species of grammatical mistake, arising from the circumstance that unlike things have names with a like inflexion. Thus, *ailing* and *cutting* have the same termination, but one applies to a state or quality, the other to an action.

II. Fallacies in Thought (*Extra Dictionem, οἱ ἔξω τῆς λέξεως*).

1. *Fallacia accidentis*, or *a dicto simpliciter ad dictum*

secundum quid, παρά τὸ συμβεβηκός. A fallacy assuming that subject and predicate have all their attributes in common. It is taking a predicate as co-extensive with a subject, when it is not.

2. *Fallacia a dicto secundum quid ad dictum simpliciter*, τὸ ἀπλῶς ἢ μὴ ἀπλῶς ἀλλὰ πῇ ἢ ποῦ ἢ ποτέ ἢ πρὸς τι λέγεσθαι, confusion of an absolute statement with a statement limited in manner, place, time, or relation.

What you bought yesterday, you eat to-day;

You bought raw meat yesterday;

You eat raw meat to-day.

This is the converse of the fallacia accidentis; many of the examples of both are instances of erroneous conversion of an universal affirmative.

3. *Ignoratio Elenchi*, τὸ παρὰ τὴν τοῦ ἐλέγχου ἀγνοίαν, an inadequate notion of confutation. A debater undertakes to contradict and overthrow a thesis, and proceeds to destroy some different position. It is the common error of arguing beside the point, of proving what has only a superficial resemblance to the conclusion, or of simply trying to distract attention from the point at issue. Mr. de Morgan classifies, along with this, any attempt to transfer the *onus probandi* to the wrong side.

4. *Fallacia consequentis, non sequitur*, τὸ παρὰ τὸ ἐπόμενον. To mistake gall for honey, because it is yellow, is a *non sequitur*. Rain wets the ground, therefore wet ground implies that it has rained. Every one in a fever is hot, but every one that is hot is not in a fever. In this case also, the examples are generally instances of wrong conversion of an universal affirmative.

5. *Petitio Principii*, τὸ παρὰ τὸ ἐν ἀρχῇ λαμβάνειν Aristotle describes five forms of this fallacy. (1) When one begs the very thing that ought to be demonstrated. (2) When one begs universally, what ought to be demonstrated particularly. (3) When one begs the particular to help to prove the universal. (4) When one begs all the particulars that compose the universal. (5) When one begs something necessarily connected with the conclusion.

Logicians discuss the question whether the syllogism itself is a *petitio principii*.

6. *Non causa pro causa*, τὸ μὴ αἷτιον ὡς αἷτιον τιθέναι, an inductive fallacy, for which another name is, *post hoc, ergo propter hoc*, which is the vice of the delusive induction called *per simplicem enumerationem*. Whitfield attributed his being

overtaken by a hailstorm on a certain occasion to his having not preached at the last town.

7. *Fallacia plurium interrogationum*, τὸ τὰ πλείω ἐρωτήματα ἐν ποιεῖν, is the fallacy of putting more questions than one as one. Why did you strike your father? It is an easy snare to ask a reason for a fact that has no existence. The first members of the Royal Society were in this predicament, when they tried to explain why a dead fish weighed more than a living fish. The answer was, it did not.

Hardly any addition has been made to Aristotle's list of Fallacies by modern writers on the Syllogism. Aristotle's principle of classification has been pronounced illogical, and new arrangements have been proposed; but his enumeration has not been materially increased.

The arrangement followed in most Manuals of Syllogistic Logic, is that adopted by Whately.

Rejecting as indistinct the division of Fallacies into those in the words (*in dictione*) and those not in the words (*extra dictionem*), Whately divides them into LOGICAL and NON-LOGICAL. The *Logical* include all cases of insufficient premises advanced as sufficient; all cases 'where the conclusion does not follow from the premises.' Such cases only, he contends, are logical in the strict sense: logic having to do only with the sufficiency of the premises given for the conclusion based upon them. As *Non-Logical* he reckons all cases where the premises are sufficient for the conclusion, 'where the conclusion does follow from the premises,' but where either the premises are unduly assumed, or the conclusion is irrelevant to the point in dispute. To settle whether the premises are legitimate or whether the conclusion is in point, passes beyond the proper sphere of Logic.

Such are Whately's main divisions. The grouping of the Aristotelian fallacies under them is as follows:—I. He subdivides *Logical* fallacies into the PURELY LOGICAL and the SEMI-LOGICAL. The *Purely Logical* are *Undistributed Middle*, and *Illicit Process* of the Major and of the Minor: two errors which Aristotle did not enumerate in his list of Fallacies (*sophismata*), whether because he considered them too palpable to be fraudulently used by a sophist, or because he had sufficiently exposed them in treating of the syllogism. The *Semi-logical* embrace all instances of ambiguous middle term. The ambiguity may be in the term itself, or may depend upon the context. The ambiguity being in the term itself, we have *Fallacia Equivo-*

cationis, and *Fallacia Amphibolias*. Our author takes an opportunity of remarking that a term may have two meanings from accident (as the term 'light'); or from some connexion of resemblance, analogy, cause and effect, &c., between the different senses. The ambiguity arising from the context, we have *Fallacia Compositionis et Divisionis*, and *Fallacia Accidentis*, and *a dicto secundum quid ad dictum simpliciter*. In these cases the middle term is not ambiguous in itself, but is used with different adjuncts in the two premises.

II. In the *Non-logical* or *Material* group, the premises may be unduly assumed, and the conclusion may be irrelevant. A premise may be altogether false and unsupported. The only guarantee against this is a knowledge of the conditions of Induction. The major premise may beg the conclusion (*petitio principii*); being either the very same as the conclusion, and differing only in form, or not quite the same as the conclusion, but unfairly implying it. So much for premises unduly assumed. Turning now to the other sub-division of the Non-logical fallacies (*ignoratio elenchi*, or *irrelevant conclusion*), we find various modes of shirking the question particularized. One way is to lay great stress upon the objections, taking no notice of what may be said in favour. Another way is to shift ground, either to something wholly irrelevant, or from one premise to another. A third way is to escape under cover of complex and general terms. And a fourth way consists in appeals to the passions and sentiments, ignoring altogether the rational grounds of the point in question. (See Book VI).

THE AXIOM OF THE SYLLOGISM.

(*Supplementary Note to the Second Edition.*)

In pp. 18, 156, 226, 237, 247, 269, the Logical Axiom of the Syllogism has been placed under the head of Inductive truth. This has not been done without misgivings, as the following remarks will show.

The drawing of a broad line between Immediate and Mediate or Syllogistic Inference, and the laying down of a Deductive Axiom founded on experience as the basis of the Syllogism, will be seen to be attended with difficulties.

The first is the anomalous middle position of the Hypo-

thetical Syllogism. If we are bound to bring hypothetical inference under one or other of the two forms, we feel that our decision is not satisfactory; the case passes somewhat beyond Immediate Inference, and yet does not reach to Syllogism.

There is the same unpleasant doubt about the cases discussed in p. 109, and p. 157, where a singular proposition has to be treated as a Universal. We cannot, without considerable straining, make these out either Equivalent propositions or Syllogisms.

The second difficulty is still greater. The question has to be raised, whether syllogistic inference is or is not Self-consistency. Is the conclusion the mere equivalent of the premises, so that to deny it, while admitting the premises, would be self-contradictory?

That the conclusion of the Syllogism flows necessarily from the premises, is generally insisted on. To refuse the conclusion would be to contradict the premises. Indeed, the self-contradiction would be as unequivocal as in the denial of an immediate inference—all A is B, some A is B. In what then consists the distinction, as regards the logical foundation, or the kind of certainty, between Mediate and Immediate inference?

In the Syllogism, the bond of necessary equivalence lies between one proposition and *two* others; in the immediate inference, it lies between one proposition and *one* other. This makes the case a degree more complicated, without apparently altering the generic character of the inference; it is an inference contained in the premises; it cannot be refused without contradiction in terms.

This circumstance of necessary, or self-consistent relationship should appear in the axiom of the Syllogism. It does so in the *dictum de omni et nullo*. That axiom seems to be a necessary truth; we feel that to deny it would be not merely to deny a fact, but to deny in one form of words what we have already affirmed in another; which expresses what is meant by 'contradiction in terms,' and by the denial of a 'necessary' truth.

The other form of the axiom—*Nota notæ*—'whatever has a mark has whatever that mark is a mark of,' must also be necessary, if it is an exact equivalent. We cannot suppose that the Syllogism under one form of axiom is an implicated or necessary inference—an analytic judgment; and, under another form, an inductive or contingent inference—a syn

thetic judgment; such a supposition could arise only from some great confusion of ideas.

• If, under the guise of *nota notæ*, the axiom is exactly equivalent in substance, as it is in appearance, to the *mathematical* axiom of mediate equality—equals of the same are equal—it would not be an axiom of self-consistency, or an analytic judgment. That axiom may be very evident, may be styled by courtesy self-evident, but it is a synthetic judgment; the subject and the predicate are not mutually implicated; its denial is not a contradiction in terms. The *subject* is 'equals of the same'—things severally compared to a common standard or measure; the *predicate* is—equal by 'coincidence,' or by being compared immediately—a totally distinct mode of comparison. These two modes are said to concur; the trial by the one mode is a test or mark of what would happen in a trial by the other mode. We have an opportunity of comparing two things with the *same third*; we have no opportunity of applying the two things to each other; we are assured by the axiom that the coincidence of the two with the common third is proof that they would coincide if we could apply them to each other. There would not be a contradiction *in terms*, there would only be a contradiction of experienced facts, if we denied that mediate coincidence infers immediate coincidence.

Mr. Mill, in the new edition of his *Logic*, Vol. I., p. 208, states that he regards Formal Logic as the logic of mere consistency, and the *dictum de omni* as its axiom; he does not insist on applying to it the *nota notæ*, although he regards that form as the proper axiom for the logic of the pursuit of truth by way of Deduction; the recognition of which can alone show how it is possible that deductive reasoning can be a road to truth. So viewed it is not self-consistency, but an inductive, contingent, or synthetic proposition, like the mathematical axiom of mediate equals.

The difference between formal deduction and *real* deduction is the difference between syllogism and inductive or experimental truth. Real deduction is the following out of an induction, and assumes the uniformity of nature. That the men living and unborn will die is a necessary inference from 'all men are mortal,' but not a necessary inference from the actual premise, which is confined to the men that have actually died. The real deduction contains *three steps*:—certain individuals possess the attributes called humanity, and also the attribute mortality; these two attributes have been

conjoined through all our past experience ; hence the presence of the one marks the presence of the other. Now, John Brown and William Smith possess the first fact, humanity, therefore they possess what it marks, that is the second fact, mortality. This is the application of the *nota notæ* in its purity and simplicity ; the uniformity of nature being supposed in addition.

For greater clearness, take another instance. 'All inert substances gravitate ;' throughout all our experience, the property 'inertness' is a mark of the property 'gravity.' Now, the etherial medium in space has the mark inertia (by resisting the comets) ; it therefore gravitates.

But still the question recurs, might not the inference in both these instances be given under the *dictum de omni* ? For, basing on the uniformity of nature, we at once convert the special observations into a general law ; men in the past have died, men in the future will die ; whence all men are mortal. Caius has the marks of man, is a man ; Caius is mortal. Inert matter gravitates ; the ether is inert ; the ether gravitates.

It would thus seem that the attainment of new truth by the way of deduction, does not imperatively demand any change of axiom. The *dictum* and the *nota notæ* are equally suitable. If so, the inference must still be a case of necessary, implied, or self-consistent truth. Of the *dictum* and the *nota notæ* alike, we must declare that their denial is a self-contradiction.

Necessary or self-consistent inference, instead of being confined to the manipulation of the equivalent forms of propositions, takes a wider sweep and embraces the Syllogism, which we should have to characterise as 'mediate self-consistency,' 'mediate necessity,' 'complex implication.' The forms lying between immediate inference or propositional equivalence, and mediate inference or syllogistic equivalence, would be regarded as incidental varieties of Self-consistency ; they need not be forced under either of the two principal genera.

When we say 'Socrates was wise,' 'Socrates was poor ;' therefore 'one man was wise and poor,' we draw a necessary or self-consistent conclusion, but not by the way of the Syllogism, as representing deductive reasoning. From 'Socrates is wise,' and 'Socrates is poor,' we can conclude 'Socrates is wise and poor ;' 'wisdom and poverty are conjoined in Socrates ;' the axiom or assumption here is—when properties can be affirmed of a subject separately, or in separate

propositions, they may be affirmed conjunctly, or in a compound proposition—which is a mode of equivalence, and cannot be denied without self-contradiction. Again, to proceed to the farther variation—one man was wise and poor—we perform the process of substituting for ‘Socrates’ the designation ‘one man,’ which properly applies to him. This is the mode of equivalence constantly assumed in working algebraic equations; where, for any expression, we insert at pleasure another equal to it. Neither of these modes is the same as the *dictum de omni*, and, therefore, they need not be forced under the syllogism, although they amount to something more than stating an equivalent form of a single proposition. The same remark applies to what has been termed ‘inference by added determinants,’ p. 109. The modes of ‘self-consistency,’ instead of conforming strictly to one or other of the two types—Equivalent propositions and Syllogism—may assume a variety of aspects.

A further discussion of the true nature of Syllogism as contrasted with real deduction will be found in *Mind*, vol. iii., p. 137. Mill’s view of syllogistic reasoning as open to the charge of *petitio principii* is examined at length, and shown to be erroneous.

